Intuitionistic Fuzzy Estimations for Similarity Queries Using Sketches of Numeric Data

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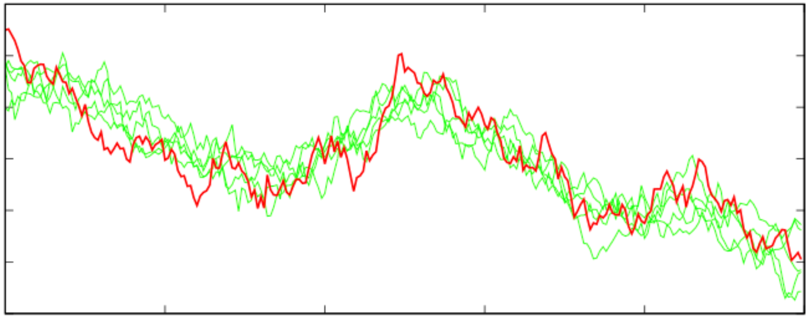
Work Overview

Context

- Consider similarity measures over time series
  - Represented as multi-dimensional numerical vectors



- Sketching for dimensionality reduction
  - Project vectors onto a space of much lower dimensionality
  - Approximately preserve Euclidean distances
  - Lower response time ←→ small accuracy loss

Work Overview

Objectives

- Intuitionistic fuzzy estimation of similarity measures
  - "is *x* similar to *q*?"
  - Truth mapped to similarity itself
  - Indefiniteness mapped to loss of accuracy
- Validation through SQL-like queries
  - Result set is intuitionistic fuzzy
  - Each row is assigned degrees of membership and non-membership

```
SELECT x
FROM tsdata
WHERE x is similar to $q
```

Random Projection

Sketching for Dimensionality Reduction

- Sketch computation

time series:
n-dimensional row vectors

transformation matrix:
n x d
$r_{i,j} = \{-1 \mid 1\}$

sketches:
d-dimensional
d << n

$\frac{1}{\sqrt{d}}$

t_1 =	$t_{1,1}$ … … … $t_{1,n}$
t_2 =	$t_{2,1}$ … … … $t_{2,n}$
……	… … …
t_m =	$t_{m,1}$ … … … $t_{m,n}$

X

r_1 =	**r_2 =**	**…**	**r_d =**
$r_{1,1}$	$r_{2,1}$	…	$r_{d,1}$
$r_{1,2}$	$r_{2,2}$	…	$r_{d,2}$
…	…	…	…
$r_{1,n}$	$r_{2,n}$	…	$r_{d,n}$

=

s_1 =	$s_{1,1}$ … $s_{1,d}$
s_2 =	$s_{2,1}$ … $s_{2,d}$
……	…
s_m =	$s_{m,1}$ … $s_{m,d}$

Random Projection

Control Parameters

*D. Achlioptas, **Database-friendly Random Projectio***
ACM SIGMOD-PODS, May, Santa Barbara, CA, 2001

Given:

ε – accuracy control parameter
β – probability control parameter
m – number of time series
n – time series dimension
d – reduced dimension

If $$d \geq \frac{4 + 2\beta}{\frac{\varepsilon^2}{2} - \frac{\varepsilon^3}{3}} \log(m)$$

Then, with probability $Pr = 1 - m^{-\beta}$

Sketches preserve the distance with distortion ε:

$$(1-\varepsilon)\left\|t_i - t_j\right\|^2 \leq \left\|s_i - s_j\right\|^2 \leq (1+\varepsilon)\left\|t_i - t_j\right\|^2$$

- Q: Is *x* similar to *y*?
  - Squared distance

$$D_{x,y} = \|x - y\|^2 = \sum_{i=1}^{n} (x_i - y_i)^2$$

  - Pearson correlation

$$C_{x,y} = \frac{1}{S_x S_y} \sum_{i=1}^{n} (x_i - M_x)(y_i - M_y)$$

    - to convert from distance

$$C_{x,y} = \frac{1}{S_x S_y} \left(\frac{Q_x + Q_y - D_{x,y}}{2} - n M_x M_y \right)$$

  - Degree of truth: C scaled to [0, 1]

$$T_{x,y} = \frac{1 + C_{x,y}}{2}$$

TS summary:

$$M_x = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$S_x = \sqrt{\sum_{i=1}^{n} (x_i - M_x)}$$

$$Q_x = \sum_{i=1}^{n} {x_i}^2$$

Intuitionistic Fuzzy Estimates

of Similarity Measures

- Q: Is *x* similar to *y*?
  - For efficiency: compute similarity on sketches *u* and *v*
  - Accuracy loss → degree of indefiniteness

$$D_{x,y}^{min} = \frac{D_{u,v}}{1+\varepsilon} \leq D_{x,y} \leq \frac{D_{u,v}}{1-\varepsilon} = D_{x,y}^{max}$$

$$C_{x,y}^{min} = \frac{1}{S_x S_y}\left(\frac{Q_x + Q_y - D_{x,y}^{max}}{2} - nM_x M_y\right) \qquad C_{x,y}^{max} = \frac{1}{S_x S_y}\left(\frac{Q_x + Q_y - D_{x,y}^{min}}{2} - nM_x M_y\right)$$

- A: $(\mu_{x,y}, \nu_{x,y})$:

$$\mu_{x,y} = \frac{1 + C_{x,y}^{min}}{2} \qquad \nu_{x,y} = \frac{1 - C_{x,y}^{max}}{2}$$

Validation

Experimental setup

Parameters:

ε (accuracy control parameter) = 0.2
β (probability control parameter) = 1
m (number of time series) = 100
n (time series dimension) = 100000
d (reduced dimension) = 1600
Pr = 0.99

PostgreSQL schema

```
CREATE TABLE data (
    id integer primary key,
    x double precision[],    -- original vector, len=100000
    mean double precision,   -- Mx, i.e. mean of x
    nstd double precision,   -- Sx, i.e. stddev w/o division by n
    sum2 double precision,   -- Qx, i.e. sum of squares
    sk double precision[]    -- sketch, len=1600
);
```

Validation

- Data preparation process
  - Generate time series synthetically from N(0, 1)
  - Generate a random matrix into a table:

```
CREATE TABLE rndmx (
    vec boolean[]    -- random vector, len=100000, {t/f} → {±1}
);                   -- contains 1600 rows
```

  - Compute sketches and store into column: ***sk***
  - Compute summaries into the additional columns:
    - *mean* (M_x), *nstd* (S_x), *sum2*(Q_x)

Validation

- Query on original time series
  - returns degree of membership

```
PREPARE sim_ts AS
select id, (1+c)/2 as mu from (
    select id, ((qsum2+dsum2-d*d)/2 - n*qmean*dmean)/(qnstd*dnstd) as c
    from (select data.id as id,
            array_length(qs.x, 1) as n, array_dist(qs.x, data.x) as d,
            qs.nstd as qnstd, qs.mean as qmean, qs.sum2 as qsum2,
            data.nstd as dnstd, data.mean as dmean, data.sum2 as dsum2
        from data qs, data
        where qs.id = $1
    ) as t
) as t;
```

Validation

- Intuitionistic fuzzy query on sketches
  - returns degrees of membership and non-membership

```
PREPARE sim_sketch AS
select id, (1+cmin)/2 as mu, (1-cmax)/2 as nu from (
    select id, ((qsum2+dsum2-d*d/0.8)/2 - n*qmean*dmean)/(qnstd*dnstd) as cmin
             , ((qsum2+dsum2-d*d/1.2)/2 - n*qmean*dmean)/(qnstd*dnstd) as cmax
    from (select data.id as id,
            array_length(qs.x, 1) as n, array_dist(qs.sk, data.sk) as d,
            qs.nstd as qnstd, qs.mean as qmean, qs.sum2 as qsum2,
            data.nstd as dnstd, data.mean as dmean, data.sum2 as dsum2
        from data qs, data
        where qs.id = $1
    ) as t
) as t;
```

Validation

- Performance improvement
  - 46 times in our setup (dim. n=100000 → d=1600)

```
ifsk2=# execute sim_ts(1);
Time: 6523.801 ms (00:06.524)
ifsk2=# execute sim_sketch(1);
Time: 141.545 ms
```

```
id |          mu          |          nu
----+----------------------+-----------------------
  1 |   0.9999999999999998 | 2.220446049250313e-16
  2 |   0.3693058085968354 |    0.420461627789663
  3 |   0.3340468220549211 |    0.443965380508609
  4 | 0.38293512934554136 |    0.4113762596527037
  5 | 0.36981105610112064 |    0.4201249993652301
  6 |   0.3383522120735153 |   0.44109817313041266
```

- Conclusion
  - Accuracy loss mapped to intuitionistic fuzzy uncertainty
  - Can be combined with other intuitionistic fuzzy estimates
    - e.g. predicates in the same IFSQL query *

* Kolev B. ***Intuitionistic Fuzzy PostgreSQL***. Advanced Studies in Contemporary Mathematics, Vol. 11, No. 2, 2005. 163-177

Kolev, B. ***Intuitionistic Fuzzy Relational Databases And Translation of the Intuitionistic Fuzzy SQL***. Proceeding of the Sixth International FLINS Conference, 1-3 Sep 2004, Blankenberge, Belgium. 189-194

Thank you for the attention!

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