



A New Parameter for Improved Assessment of the Tracing Repeatability for Area Measurements of the Left Ventricle

Wai-Tak (Arthur) Hung

Cancer Institute NSW
Level 1, Biomedical Building
Australian Technology Park
Eveleigh NSW 2015, Australia
E-mail: arthur_hung@hotmail.com

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Abstract: In order to validate an automated border detection (ABD) algorithm for obtaining a cross-sectional area of the left ventricle of heart, the areas obtained by ABD need to be compared with the areas obtained by a trained tracer as the latter's measurements are regarded as gold standard. Before performing an agreement analysis between the areas obtained by the ABD and that of the tracer, we need to know how well the tracer can repeat the tracing of the same image. A new parameter is proposed to assess the repeatability of this tracing. The results of three sets of 30x2 measurements have been analyzed. It was found that this new parameter can reveal the true difference in two tracings of the same image both spatially and numerically on a scale of 0 to 1. It can also yield the area differences between the ABD and tracers.

Keywords: Validation, Left-ventricle, Repeatability, Area tracing.

Introduction

It is important to perform a repeatability study when any two methods are compared because if one or both of the methods has poor repeatability, the agreement between the two methods is also bound to be poor. Furthermore, replicated measurements on a series of subjects are needed to examine repeatability. Suppose two replicate measurements (X_{1j} and X_{2j}) are obtained for subject j . We can plot the difference, $d_j = X_{1j} - X_{2j}$, against the mean, $m_j = (X_{1j} + X_{2j})/2$ [1]. From the plot we can see whether the within-subject repeatability (as measured by the difference) is associated with the size of the measurements. The definition of repeatability coefficient adopted by the British Standards Institution is twice the standard deviation of the differences, that is, $2s$ where

$$s = \left[\sum_{j=1}^n (X_{1j} - X_{2j})^2 / n \right]^{1/2} .$$

With this statistical measure it would be expected that 95% of the differences would be less than $2s$.

In [2] is used the above approach in the validation process when they compared the area obtained by the ABD and that of the tracer for transesophageal echocardiograms. It should be noted that



the use of the difference based measure is acceptable for the one-dimensional type of measurements. However, area is two-dimensional and the difference measure for the two areas conveys only the size difference with no information about the difference in position. Therefore, zero difference in area does not mean that the two tracings are exactly the same. The two traced areas can be of far different shape but can still have zero difference as long as the traced areas have numerically the same value. Fig. 1 gives an example of having similar traced areas but the spatial positions of the two traced areas are very different. Therefore, a new parameter is required to compensate for this serious weakness and we now describe such a parameter.

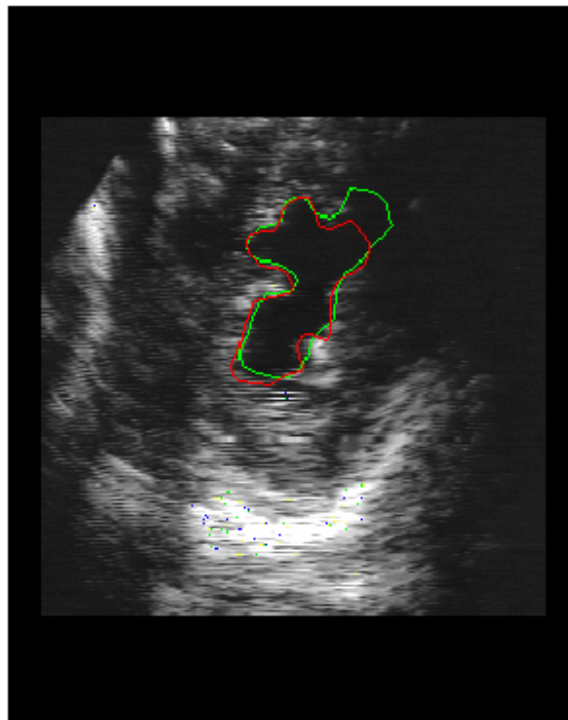


Fig. 1 A sample tracing

Method

Beside using the values of the traced areas, X_{1j} and X_{2j} , we also obtained the intersection area of the two traced areas, denoted by I_j for subject j , and let U_j be the largest area covered by the two traced areas for the same subject, that is,

$$U_j = X_{1j} + X_{2j} - I_j.$$

It should be noted all values of area are in number of pixels. We then introduce a parameter, m'_j , to estimate the true area traced for subject j as:

$$m'_j = \frac{U_j + I_j}{2} = \frac{X_{1j} + X_{2j}}{2},$$



which, of course, is m_j . We also define a new parameter, d'_j , to estimate the tracing error as:

$$d'_j = U_j - m_j = \frac{1}{2}(U_j - I_j).$$

When d'_j is zero, it means that the two traced areas are identical and at the same position. Also, $d'_j = m_j$ when the two tracings are disjoint, that is, without overlapping or $I_j = 0$.

In fact, the above concept can be presented in terms of set theory. This new form can accommodate more than 2 replicates. Let M_{ij} be the i^{th} matrix of size $u \times v$ containing values of 1 for the enclosed area and 0 otherwise for subject j . Then d'_j can be written as:

$$d'_j = \frac{1}{t} \text{Area} \left\{ \bigcup_{i=1}^t M_{ij} - \bigcap_{i=1}^t M_{ij} \right\},$$

where t is the number of replicates. Also, let re-define U_j and I_j as:

$$U_j = \bigcup_{i=1}^t M_{ij} \quad \text{and} \quad I_j = \bigcap_{i=1}^t M_{ij}.$$

A fast method for calculating U_j and I_j is proposed. Again let M_{ij} be the i^{th} matrix of size $u \times v$ containing values of 1 for the enclosed area and 0 otherwise for subject j . Let T_{ijpq} be the element of the matrix M_{ij} .

$$T_{j pq} = \sum_{i=1}^t T_{ijpq}, \quad U_{j pq} = \begin{cases} 0 & T_{j pq} = 0 \\ 1 & 0 < T_{j pq} \leq t \end{cases} \quad \text{and} \quad I_{j pq} = \begin{cases} 1 & T_{j pq} = t \\ 0 & T_{j pq} \neq t \end{cases}.$$

Therefore,

$$U_j = \sum_{p=1}^u \sum_{q=1}^v U_{j pq} \quad \text{and} \quad I_j = \sum_{p=1}^u \sum_{q=1}^v I_{j pq}.$$

Results

The results of three sets of 30×2 measurements each have been analyzed. A sample set of data is provided in Table 1. In this table, $|d|$ means the absolute value of d . Three graphs are shown in Fig. 2, Fig. 3 and Fig. 4. Fig. 2 is a scatter plot of d against m .



Table 1. Sample Data

Subject	X_{1j}	X_{2j}	I_j	d_j	$ d_j $	d'_j	m_j
1	10059	10053	9707	6	6	349	10056
2	4959	3130	3077	1829	1829	967,5	4044,5
3	8592	9149	7928	-557	557	942,5	8870,5
4	2774	6084	2767	-3310	3310	1662	4429
5	10517	10132	9995	385	385	329,5	10324,5
6	5857	5684	5512	173	173	258,5	5770,5
7	4569	4080	3757	489	489	567,5	4324,5
8	3306	3106	2881	200	200	325	3206
9	2713	2985	2534	-272	272	315	2849
10	4164	3733	3632	431	431	316,5	3948,5
11	2712	2613	2426	99	99	236,5	2662,5
12	4021	3834	3714	187	187	213,5	3927,5
13	3895	3968	3569	-73	73	362,5	3931,5
14	4081	3640	3575	441	441	285,5	3860,5
15	6405	5238	5056	1167	1167	765,5	5821,5
16	3796	3928	3574	-132	132	288	3862
17	5864	5681	5489	183	183	283,5	5772,5
18	6427	6253	5550	174	174	790	6340
19	4493	4628	4230	-135	135	330,5	4560,5
20	3953	4148	3714	-195	195	336,5	4050,5
21	7352	6694	6630	658	658	393	7023
22	4018	4282	3753	-264	264	397	4150
23	4283	3847	3728	436	436	337	4065
24	3016	3047	2803	-31	31	228,5	3031,5
25	3428	4114	3218	-686	686	553	3771
26	3497	3629	3269	-132	132	294	3563
27	10368	10000	9604	368	368	580	10184
28	9031	11321	8857	-2290	2290	1319	10176
29	6978	4301	4301	2677	2677	1338,5	5639,5
30	10662	10326	9915	336	336	579	10494

Fig. 3 is another scatter plot showing d' against m . The combined set of data having both $|d|$ and d' against m is shown in Fig. 4. In some cases the d' values are very large when their corresponding d values are close to zero. Table 2 summarizes the repeatability results of the three sets of data. In Table 2, the new standard deviation, s' and the two average percentage relative errors, \bar{e} and \bar{e}' , are respectively defined as:

$$s' = \left[\frac{1}{n} \sum_{j=1}^n (d'_j)^2 \right]^{1/2},$$



$$\bar{e} = \frac{1}{n} \left[\sum_{j=1}^n \frac{d_j}{m_j} \right] \times 100\%$$

and

$$\bar{e}' = \frac{1}{n} \left[\sum_{j=1}^n \frac{d'_j}{m_j} \right] \times 100\% .$$

It should be noted that the equation for \bar{e}' , the term d'_j/m_j stands for the relative error for the tracing on subject j . It is zero when the two traced areas are identical in shape and location. It will give the value 1 when the two traced areas do not intersect each other. Therefore, we have a *scale* from zero to 1 (or 0% to 100%) to assess the repeatability performance on each pair of tracing, whereas the term d_j/m_j in the equation for \bar{e} does not have this useful scale feature. It is also noted that the parameter, s' , might not be a good indicator for performance comparison because it depends on the average area size. It is suggested that \bar{e}' is better because it is an average of the normalized estimated error values. An alternative calculation using the new approach is to divide s' by the mean estimated areas of all traces as a measure for comparing the performance of tracers. The results give the same ranking as \bar{e}' showing in Table 2.

Table 2. Analysis summary showing the advantage of the new parameter d' in detecting differences

Data Set	s.d. and average % error values detected			
	s	s'	\bar{e} (%)	\bar{e}' (%)
1	2227,78	1222,57	22,18	16,28
2	2404,61	1380,81	16,39	14,63
3	1015,10	645,35	11,87	10,39

It should be noted that there is no point in comparing \bar{e} and \bar{e}' because they are different in scale. The former has the range from 0% to 200% while the latter has the upper limit of 100%. Also, there is no single percentage value that can be used to tell whether the repeatability fails or not. It is different from study to study and it all depends on clinical acceptance.

Conclusion

The new parameter, i.e., the average percentage relative errors (\bar{e}'), is recommended for the analysis of repeatability of area tracing. Furthermore, it can also be used to detect the area difference between the one obtained by the ABD and that from the tracer when we perform the agreement analysis of the different methods.

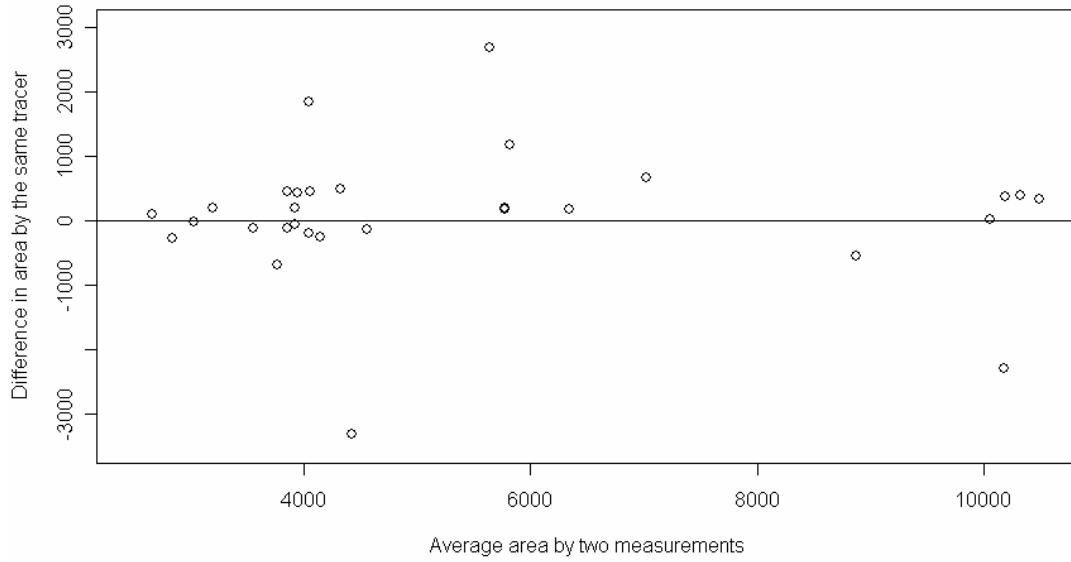


Fig. 2 d against m

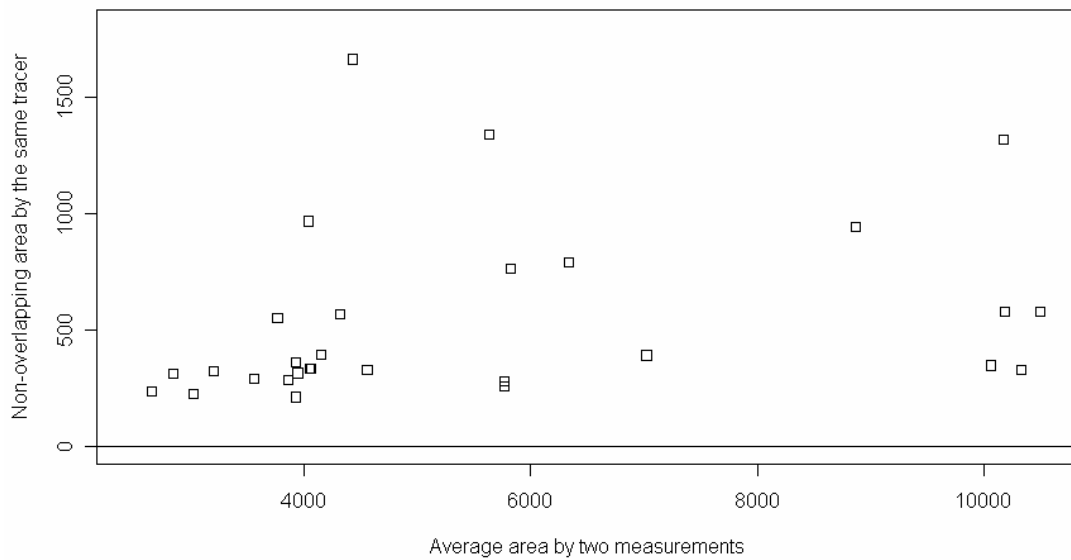


Fig. 3 d' against m

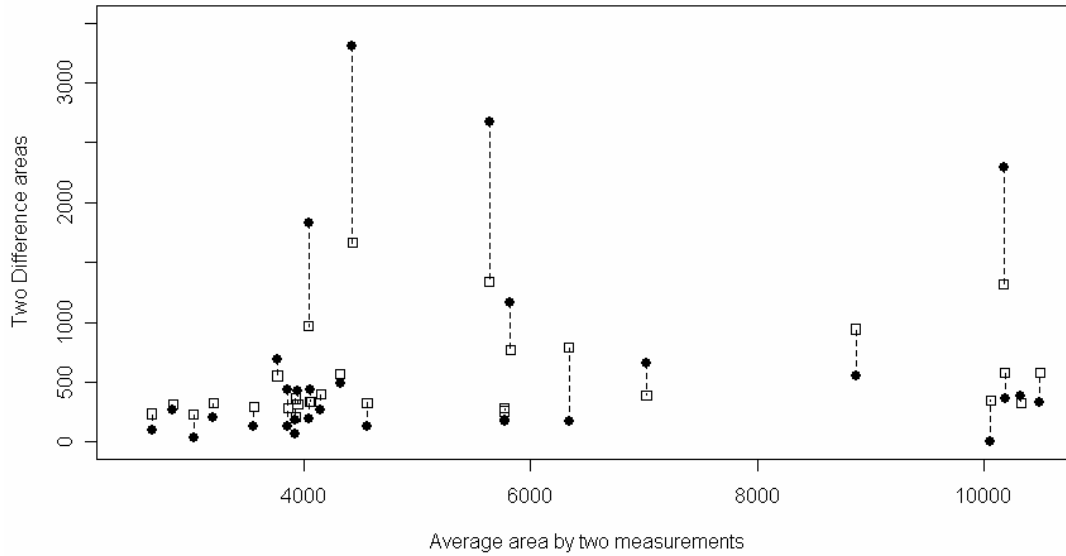


Fig. 4 A comparison between $|d|$ (●) and d' (□)

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