

# Modelling and Extremum Seeking Control of a Cascade of Two Anaerobic Bioreactors

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**Abstract:** The principle of extremum seeking control has been applied on a cascade of two anaerobic bioreactors using the dilution rate as control action and the biogas flow rates as measured outputs to be maximized. In all cases maximum biogas flow rate with sensible decrease of the general output depollution parameter (compared to the case of one single bioreactor) has been obtained, starting from different initial conditions. With the same algorithm, good performances have been obtained in the presence of variations of the inlet organics. Its implication for biotechnology may result in substantial economic benefits.

**Keywords:** Anaerobic digestion, Cascade of bioreactors, Nonlinear model, Steady-state analysis, Extremum seeking control.

## Introduction

In the anaerobic digestion (AD) of organic wastes, the organic matter is decomposed by microorganisms into biogas and compost in the absence of oxygen. Generally these processes are carried out in continuously stirred tank bioreactors (BR) [3].

AD is an effective biotechnological process for treatment of different agricultural, municipal and industrial wastes. It combines environmental depollution (ecological aspect) with production of renewable energy – biogas, the main component of which is methane (energetical aspect) [3].

Recently, AD in interconnected (cascade) bioreactors has manifested some advantages concerning wastes degradation and biogas productivity [5, 11].

A lot of AD models are known. All of them present extreme (maximal) characteristic concerning the biogas flow rate via the dilution rate [9, 10].

Recently, two methods for optimal control of bioprocesses have been demonstrated in some applications – self-optimizing control and extremum seeking (ES) control [4]. The task of ES control is to find the operating set-points that maximize or minimize an objective function [1]. Its implication for biotechnology may result in substantial economic benefits.

In the last two decades some new results concerning the ES control of nonlinear systems have been obtained [2, 6-8, 12]. Two approaches of ES control for bioreactors are known – model-

based approach and nonmodel-based approach. However, these algorithms for optimal control have not yet been applied for cascades of BR.

The aim of this paper is to investigate an algorithm for ES control of a cascade of two anaerobic bioreactors (Fig. 1) using the dilution rate of the first bioreactor as control action and the biogas flow rate of the first bioreactor as measured output to be maximized. The control algorithm has been tested on a nonlinear model with Monod type of kinetics for both bioreactors.

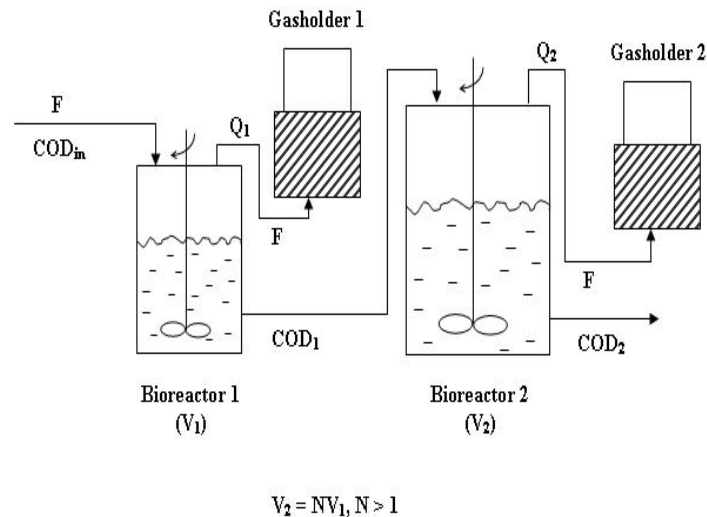


Fig. 1 Principle scheme of a cascade of two anaerobic bioreactors

## Process modeling

### Experimental studies

Laboratory experiments of AD of mixture of activated sludge (70%) and milk whey (30%) with  $COD = 10.2 \text{ gO/dm}^3$  are carried out in a cascade of two anaerobic bioreactors (with working volumes of 2 and 14  $\text{dm}^3$ , respectively) at a mesophilic temperature of 34°C. As an example, some results concerning the production of biogas and depollution effect ( $COD$ ) in steady-states are shown in Table 1 [11].

Table 1. Some experimental results

$D_1$ ( $\text{day}^{-1}$ )	$Q_1$ ( $\text{dm}^3 \cdot \text{day}^{-1}$ )	$COD_1$ ( $\text{gO} \cdot \text{dm}^{-3}$ )	$D_2$ ( $\text{day}^{-1}$ )	$Q_2$ ( $\text{dm}^3 \cdot \text{day}^{-1}$ )	$COD_2$ ( $\text{gO} \cdot \text{L}^{-1}$ )
0.025	0.46	2.20	0.0036	0.20	0.80
0.050	1.00	2.76	0.0072	0.25	0.92

From the experimental data shown in Table 1 one may conclude that the total yield of biogas is about 25% more and the depollution effect is about three times better than in a single BR [9].

### Mathematical modelling

The model of a cascade of two anaerobic bioreactors, used in this paper, has been developed on the basis of flux balance [11].

For  $BR_1$  the model is based on a three-stage reaction scheme and includes equations for the hydrolysis (1) of the soluble organic matter with concentration  $S_0$  in  $BR_1$  for influent waste with concentration  $S_{0i}$  and dilution rate  $D_1$ , the growth of acidogenic bacteria (2) with

concentration  $X_{11}$  and methanogenic bacteria (4) with concentration  $X_{21}$ , the dynamics of the substrate (with concentration  $S_{11}$ ) for acidogenic bacteria (3) and of acetate (with concentration  $S_{21}$ ) production (5), the formation of biogas with flow rate  $Q_1$  (6) and the equation for  $COD_1$  of the outlet of BR<sub>1</sub> (7):

$$\frac{dS_0}{dt} = -D_1 S_0 - \beta X_{11} S_0 + D_1 Y_p S_{0i}, \quad S_0 > 0 \quad (1)$$

$$\frac{dX_{11}}{dt} = (\mu_{11} - D_1) X_{11}, \quad X_{11} > 0 \quad (2)$$

$$\frac{dS_{11}}{dt} = -D_1 S_{11} + \beta X_{11} S_0 - \mu_{11} \frac{X_{11}}{Y_{11}}, \quad S_{11} > 0 \quad (3)$$

$$\frac{dX_{21}}{dt} = (\mu_{21} - D_1) X_{21}, \quad X_{21} > 0 \quad (4)$$

$$\frac{dS_{21}}{dt} = -D_1 S_{21} + Y_{B1} \mu_{11} X_{11} - \mu_{21} \frac{X_{21}}{Y_{21}}, \quad S_{21} > 0 \quad (5)$$

$$Q_1 = Y_{g1} \mu_{21} X_{21} \quad (6)$$

$$COD_1 = c_{11} S_0 + c_{21} S_{11} + c_{31} S_{21}, \quad COD_1 > 0 \quad (7)$$

In Eq. (7)  $c_{i1}$  ( $i = 1, 2, 3$ ) are conversion factors from  $\text{g.dm}^{-3}$  to  $\text{gO.dm}^{-3}$  for BR<sub>1</sub>.

For the specific growth rates of bacteria in BR<sub>1</sub>, Monod type (including decay coefficients  $k_1$  and  $k_2$ ) nonlinear functions have been adopted:

$$\mu_{11} = \frac{\mu_{11\max} S_{11}}{K_{S11} + S_{11}} - k_1, \quad \mu_{11} > 0 \quad (8)$$

$$\mu_{21} = \frac{\mu_{21\max} S_{21}}{K_{S21} + S_{21}} - k_2, \quad \mu_{21} > 0 \quad (9)$$

The dilution rate for BR<sub>1</sub> is:

$$D_1 = \frac{F}{V_1} > 0 \quad (10)$$

For BR<sub>2</sub> the model is based on a two-stage reaction scheme (the hydrolysis equation is not included because supposed to have taken place only in BR<sub>1</sub>) and includes equations for the growth of acidogenic (11) and methanogenic (13) bacteria (with concentrations  $X_{12}$  and  $X_{22}$  respectively), the degradation of the not completely digested in BR<sub>1</sub> soluble organic matter (12) and of the acetate (14), production of biogas with flow rate  $Q_2$  (15) and the equation for the depollution parameter  $COD_2$  of the effluent of BR<sub>2</sub> (16):

$$\frac{dX_{12}}{dt} = \mu_{12} X_{12} + D_2 (X_{11} - X_{12}), \quad X_{12} > 0 \quad (11)$$

$$\frac{dS_{12}}{dt} = D_2 (S_{11} - S_{12}) - \mu_{12} \frac{X_{12}}{Y_{12}}, \quad S_{12} > 0 \quad (12)$$

$$\frac{dX_{22}}{dt} = \mu_{22} X_{22} + D_2 (X_{21} - X_{22}), \quad X_{22} > 0 \quad (13)$$

$$\frac{dS_{22}}{dt} = D_2 (S_{21} - S_{22}) + Y_{B2} \mu_{22} X_{12} - \mu_{22} \frac{X_{22}}{Y_{22}}, \quad S_{22} > 0 \quad (14)$$

$$Q_2 = Y_{g2} \mu_{22} X_{22} \quad (15)$$

$$COD_2 = c_{22}S_{12} + c_{32}S_{22} + c_{42}S_{11} + c_{52}S_{21}, \quad COD_2 > 0 \quad (16)$$

In (16)  $c_{i2}$  ( $i = 2, 3, 4, 5$ ) are conversion factors from  $g.dm^{-3}$  to  $gO.dm^{-3}$  for  $BR_2$ .

For the specific growth rates of bacteria in  $BR_2$ , Monod type nonlinear functions have been adopted as well:

$$\mu_{12} = \frac{\mu_{12max}S_{12}}{K_{S12} + S_{12}}, \quad \mu_{12} > 0 \quad (17)$$

$$\mu_{22} = \frac{\mu_{22max}S_{22}}{K_{S22} + S_{22}}, \quad \mu_{22} > 0 \quad (18)$$

The dilution rate for  $BR_2$  is:

$$D_2 = \frac{F}{V_2} > 0 \quad (19)$$

Taking into account that the volume of  $BR_2$  is  $N$  times bigger (in our case  $N = 7$ ) than that of  $BR_1$ , the following ratio between dilution rates has been obtained:

$$D_2 = \frac{F}{NV_1} = \frac{D_1}{N}, \quad (N > 1) \quad (20)$$

For a single anaerobic bioreactor, the input-output static characteristics  $Q = f(D)$  and  $COD = f(D)$  have been obtained, using Symbolic toolbox of Matlab and the model (1) to (9). They are shown on Fig. 2 [9]. The static characteristic  $Q = f(D)$  is a family of nonlinear curves (with parameter  $S_{0i}$ ), presenting a maximum. It is possible to obtain analytically values for  $D_{max}$  (corresponding to the maximal value of  $Q - Q_{max}$ ) and for the technological bound  $D_{sup}$  (related with wash-out of bacteria). The static characteristic  $COD = f(D)$  is a unique (non-depending of  $S_{0i}$ ) nonlinear curve.

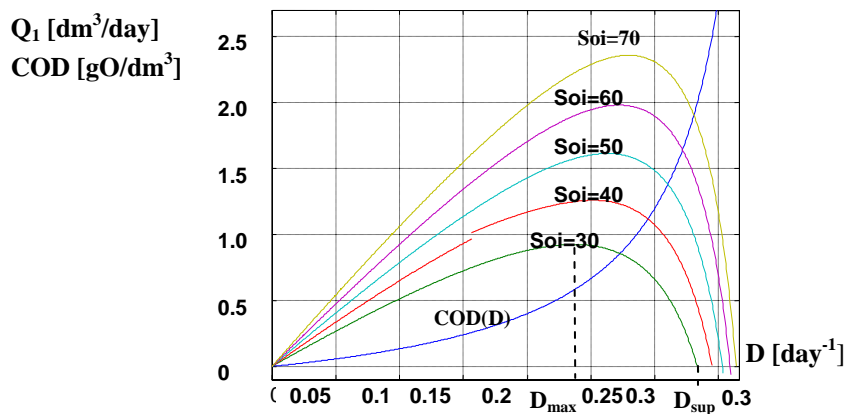


Fig. 2 Input-output characteristics

From Fig. 2 one may conclude that for small values of  $D$  ( $D \ll D_{max}$ ) the depollution effect is much better and the input-output static characteristics are closer to linear. In  $BR_1$  the aim is to maximize the biogas flow rate ( $D_1$  close to  $D_{max}$ ) and the depollution effect will be neglected.  $BR_2$  will operate with small values of  $D_2$  ( $D_2 \ll D_{max}$ ) – in our case 7 times lower and the

depollution effect is much better than in BR<sub>1</sub>. However, the biogas flow rate of BR<sub>2</sub> is significant as well due to the greater volume of this bioreactor.

The measurable outputs are  $Q_1$  and  $Q_2$ . However, only  $D_1$  is the control input.

It is very difficult to obtain the exact coefficients values [11]. That is why for this study the following arbitrary coefficients have been adopted:

$$\begin{aligned}\mu_{11max} &= \mu_{12max} = 0.22 \text{ day}^{-1}, \quad \mu_{21max} = \mu_{22max} = 0.25 \text{ day}^{-1}, \\ K_{S11} &= K_{S12} = 1.6 \text{ g.dm}^{-3}, \quad K_{S21} = K_{S22} = 1.6 \text{ g.dm}^{-3}, \\ k_1 &= k_2 = 0.02 \text{ day}^{-1}, \quad \beta = 3.0, \\ Y_{11} &= Y_{12} = 0.15, \quad Y_{21} = Y_{22} = 0.24, \\ Y_p &= 0.144, \quad Y_{B1} = Y_{B2} = 5.0, \quad Y_{g1} = Y_{g2} = 4.35.\end{aligned}$$

## Extremum seeking control

### *Problem statement*

Let us assume that the goal of the AD process in BR<sub>1</sub> is the production of biogas. As an optimization objective, it is then natural to consider the maximization of the biogas flow rate  $Q_1$  [ $\text{dm}^3 \cdot \text{day}^{-1}$ ]:

$$Q_1 \Rightarrow \text{Max} \quad (21)$$

In the next paragraph we show that the steady states of the AD process in BR<sub>1</sub> are characterized by a non-monotonic map relating the biogas flow rate  $Q_1$  (controlled output) to the dilution rate  $D_1$ , which is our control input. The purpose of the extremum seeking method is then to iteratively adjust the dilution rate in order to steer the process to the maximum of this map.

### *Steady-state analysis of the open-loop system*

In ideal stationary conditions, all the derivatives in the model (1)-(9) are equal to zero. Thus the following static model is obtained:

$$-D_1 S_0 - \beta X_{11} S_0 + D_1 Y_p S_{0i} = 0 \quad (22)$$

$$(\mu_{11} - D_1) X_{11} = 0 \quad (23)$$

$$-D_1 S_{11} + \beta X_{11} S_0 - \mu_{11} \frac{X_{11}}{Y_{11}} = 0 \quad (24)$$

$$(\mu_{21} - D_1) X_{21} = 0 \quad (25)$$

$$-D_1 S_{21} + Y_{B1} \mu_{21} X_{11} - \mu_{21} \frac{X_{21}}{Y_{21}} = 0 \quad (26)$$

$$Q_1 = Y_{g1} \mu_{21} X_{21} \quad (27)$$

From (22) to (28) and taking into account (8) and (9) we obtain:

$$S_0^* = \frac{D_1 Y_p S_{0in}}{D_1 + \beta X_{11}} \quad (28)$$

$$S_{11}^* = \frac{k_{S11} D_1}{\mu_{11m} - D_1} \quad (29)$$

$$S_{21}^* = \frac{k_{S_{21}} D_1}{\mu_{21m} - D_1} \quad (30)$$

$$X_{11}^* = Y_{11} \left( S_{0i} - \frac{k_{S_{11}} D_1}{\mu_{11max} - D_1} \right) \quad (31)$$

$$X_{21}^* = Y_{21} \left( \frac{k_{S_{11}} D_1}{\mu_{21max} - D_1} \right) + Y_{B1} Y_{11} X_{11} \quad (32)$$

$$Q_1 = Y_{g1} D_1 X_{21}^* \quad (33)$$

$$COD_1 = c_{11} S_0^* + c_{21} S_{11}^* + c_{31} S_{21}^* \quad (34)$$

From (31), (32) and (33) we obtain (by Symbolic toolbox of Matlab) algebraic expressions for  $D_{sup1}$ ,  $D_{sup2}$  and  $Q_1$ . The equilibrium for this model is defined only for  $D^* < D_{sup1}^*$  [8] and it is stable for all values of  $D_1^*$  for which it is defined. For the above model we obtain:

$$\text{for } S_{0i} = 65 \text{ g.dm}^{-3} \Rightarrow Q_{1max} = 0.47, D_{1max} = 0.10, D_{sup}^* = 0.15;$$

$$\text{for } S_{0i} = 75 \text{ g.dm}^{-3} \Rightarrow Q_{1max} = 0.59, D_{1max} = 0.108, D_{sup}^* = 0.17.$$

### *A peak seeking control via the dilution rate*

The peak seeking feedback scheme is shown on Fig. 3. Its basic idea is to employ periodic excitation signal  $asin(\omega t)$ , which is added to the signal  $\hat{D}$ . If this excitation signal is slow, then the AD process appears as a static map  $Q = Q(D)$  and its dynamics does not interfere with the peak-seeking scheme. If  $\hat{D}$  is on either side of  $D_{max}$ , the excitation signal  $asin(\omega t)$  creates a periodic response of  $Q$ , which is either in phase or out of phase with  $asin(\omega t)$ . The high-pass filter  $s/(s+\omega_h)$  eliminates the “DC component” of  $Q$  [6]. Thus,  $asin(\omega t)$  and  $\{s/(s+\omega_h)\}Q$  will be (approximately) two sinusoids, which are: in phase for  $\hat{D} < D_{max}$  or out of phase for  $\hat{D} > D_{max}$ . In either case, the product of two sinusoids will have a “DC component”  $\xi$ , that can be argued to be approximately the sensitivity function  $(a^2/2)[Q(D)](\hat{D})$ . Then the integrator  $\hat{D} = (k/s)\xi$  is approximately the gradient update law:

$$\frac{d}{dt} \hat{D} = k(a^2/2) \frac{d}{dt} [Q(D)](\hat{D}) \quad (35)$$

driven by the sensitivity function which tunes  $\hat{D}$  to  $D_{max}$ .

The tuning parameters in this scheme  $\omega_h$ ,  $\omega$ ,  $a$  and  $k$  must be chosen as follows [6, 12]:

$$O(1) \gg \omega \gg \omega_h, a, k \quad (36)$$

where  $O(1)$  is speed of nonlinear dynamics,  $\omega$  and  $a$  are frequency and amplitude of the excitation signal, respectively,  $\omega_h$  and  $k$  are parameters of the high-pass filter and the integrator in the peak seeking scheme.

Thus, the overall feedback system has three time scales:

1. Fastest – the process (with the stabilizing controller);
2. Medium – the periodic excitation signal;
3. Slow – the filter in the peak-seeking scheme.

As a result, this peak seeking control is model-free and able to automatically tune the dilution rate in the right direction. The scheme shown on Fig. 3 guarantees the stability result outlined in the following theorem:

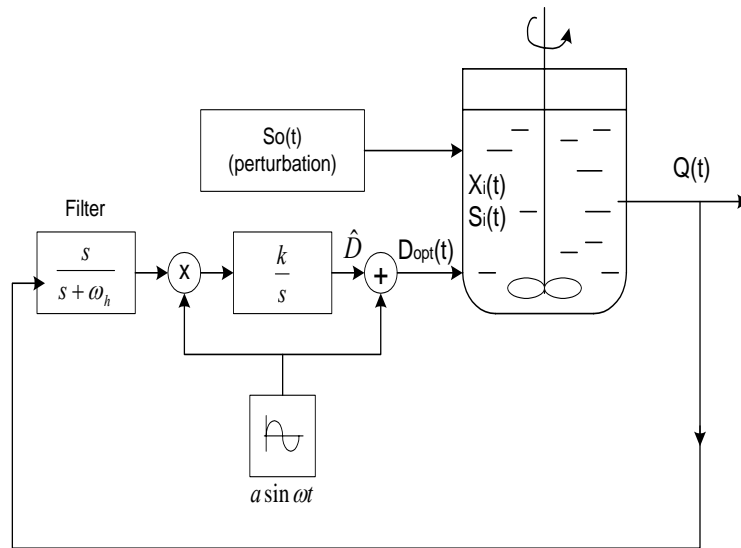


Fig. 3 The peak seeking feedback scheme

**Theorem:** Consider the feedback system on Fig. 3 and assume that the AD dynamic model has the following properties:

1. For  $D$  in an interval  $[D', D'']$  there is an isolated one-dimensional manifold of equilibrium  $E(D)$  which depends smoothly on  $D$ .
2. Each equilibrium in  $E[D', D'']$  is exponentially stable with a  $O(1)$  rate of decay.
3. The equilibrium value of the output  $Q$  on  $E[D', D'']$  is a smooth function of  $D$  with a maximum at  $D = D_{max}$ .

Then there exists a ball of initial conditions around the equilibrium corresponding to  $D = D_{max}$  and a positive constant  $\bar{\omega} \ll 1$  such that for all  $\omega \in (0, \bar{\omega})$  and all  $a, k, \omega_h \ll \omega$ , the solution converges to a  $O(\omega)$  neighborhood of that equilibrium.

This theorem is an interpretation for AD process of the more general result for continuous type of biotechnological processes [12] with detailed proof in [6].

### Simulation results

Our purpose is to tune  $D_1$  to  $D_{1max}$ . We implement the peak seeking scheme with the following choice of parameters (obtained heuristically):  $\omega_h = 0.009$ ;  $\omega = 0.08$ ;  $a = 0.01$ ;  $k = 0.018$ . As we don't know the real initial conditions, we will investigate both possibilities – starts from left and right of the maxima. First, we start from an initial dilution rate  $D(0) = 0.034 \text{ day}^{-1}$  lower than the optimal rate  $D_{max}$ . The time responses of the output  $Q_1$  and of the resulting total amount of biogas  $Q$ , defined as a sum of the biogas flow rates of BR<sub>1</sub> and BR<sub>2</sub> ( $Q = Q_1 + Q_2$ ) are shown on Fig. 4. The time responses of the control parameter  $D_1$  and the resulting  $D_2$  are shown on Fig. 5. Time responses of  $COD_1$  and the resulting  $COD_2$  are shown on Fig. 6. The maximum seeking process in the phase plane  $Q_1 - D_1$  is shown on Fig. 7. In the second simulation study we start from an initial dilution rate  $D(0) = 0.11 \text{ day}^{-1}$  larger than the optimum value  $D_{max}$ . Time responses as for the previous case are shown on

Fig. 8 to Fig. 11, respectively. In all cases for  $t = 400$  days, a step variation of  $S_{0i}$  occurs (from  $65 \text{ g/dm}^3$  at  $75 \text{ g/dm}^3$ , e.g. increase with about 15%).

In both cases the peak seeking approaches the appropriate peak (for  $S_{0i} = 65 \text{ g.dm}^{-3}$  and  $S_{0i} = 75 \text{ g.dm}^{-3}$ , respectively). From all figures it is clear that the settling time is about 300 days (in real conditions) and the improvement in performance to the maximum output is about 200%. This means the performance is improved with a rate of about 0.7% per day. This rate of improvement is satisfactory but it is certainly not impressive. The convergence to the peak can be made faster by tuning the parameters of the scheme and by introducing an appropriate phase shift in the perturbation sinusoid. However, if we choose parameters, which make the convergence from the left side of the peak faster, they are too aggressive for the right side of the peak and may lead to instability. As we do not assume to know the location of the peak, the adaptation must proceed cautiously. The oscillations of the output  $Q_1$  in Fig.4 are about  $\pm 3\%$  of the peak equilibrium value of  $Q_{1max}$ , while the oscillations of  $D_1$  on Fig. 5 are about 10% from  $D_{1max}$ . These results are completely satisfying, regarding the real experimental data [11].

It is evident from Fig. 4 and Fig. 8 that the total biogas flow rate  $Q$  is higher than that for one single bioreactor ( $Q_1$ ) with sensible decreasing of the general output depollution parameter  $COD_2$  (Fig. 6 and Fig. 10).

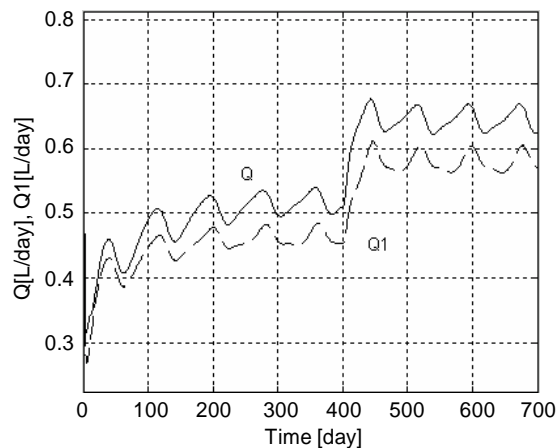


Fig. 4 Time response of  $Q_1$  and  $Q = Q_1 + Q_2$  with initial condition  $D_1(0) = 0.034 \text{ day}^{-1}$

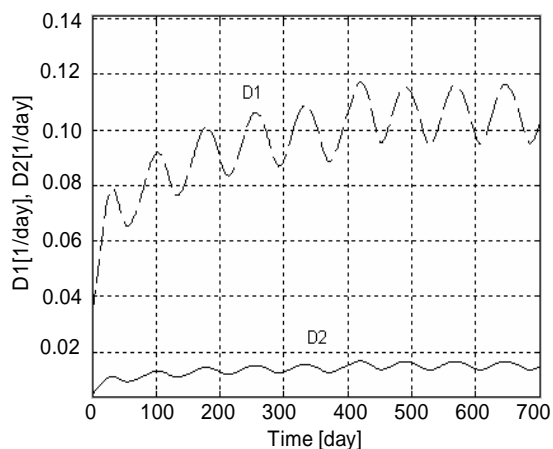


Fig. 5 Time response of  $D_1$  and  $D_2$  with initial condition  $D_1(0) = 0.034 \text{ day}^{-1}$



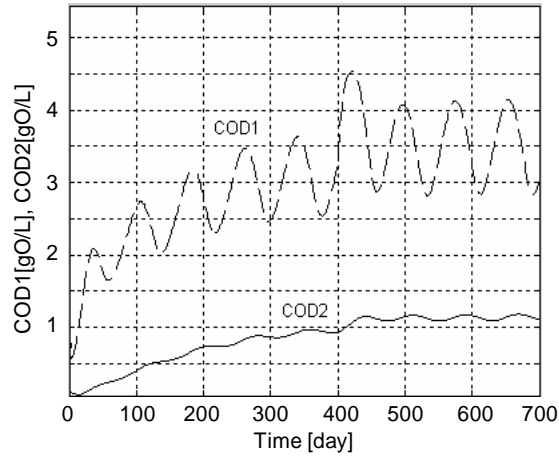


Fig. 6 Time response of  $COD_1$  and  $COD_2$  during the extremum seeking process for  $BR_1$  with initial condition  $D_1(0) = 0.034 \text{ day}^{-1}$

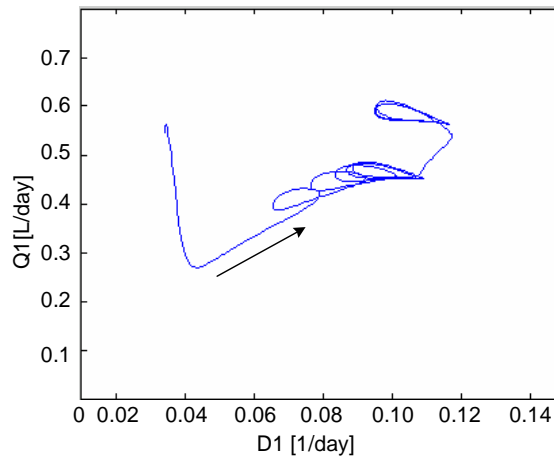


Fig. 7 The maximum seeking process in the phase plane  $Q_1 - D_1$  with initial condition  $D_1(0) = 0.034 \text{ day}^{-1}$

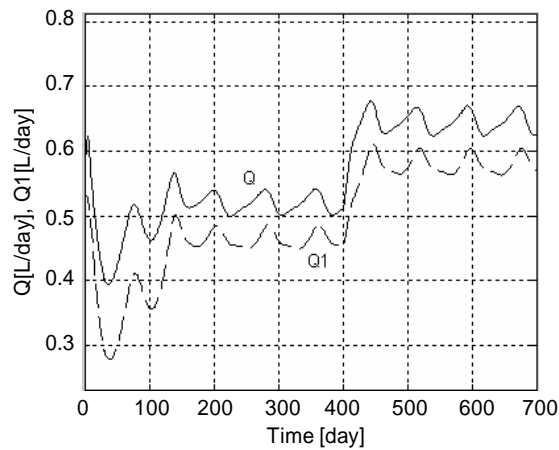


Fig. 8 Time response of  $Q_1$  and  $Q = Q_1 + Q_2$  with initial condition  $D_1(0) = 0.11 \text{ day}^{-1}$

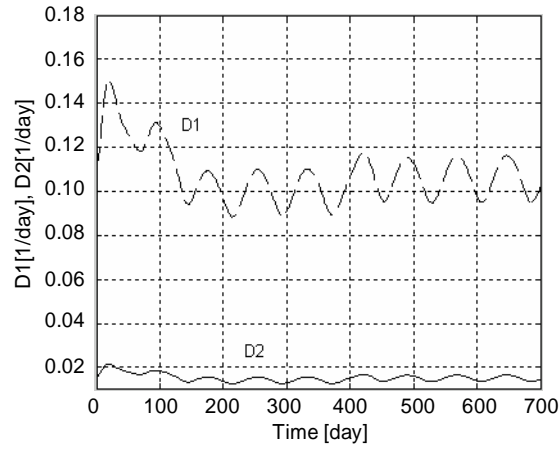


Fig. 9 Time response of  $D_1$  and  $D_2$  with initial condition  $D_1(0) = 0.11 \text{ day}^{-1}$

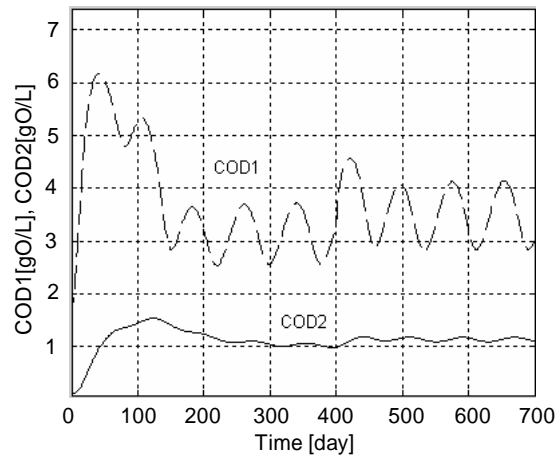


Fig. 10 Time response of  $COD_1$  and  $COD_2$  during the extremum seeking process for  $BR_1$  with initial condition  $D_1(0) = 0.11 \text{ day}^{-1}$

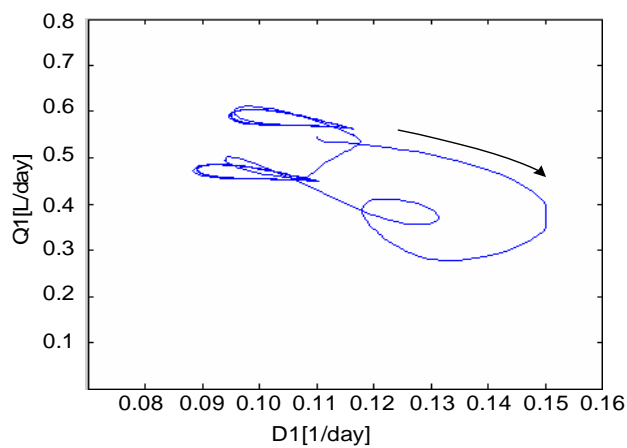


Fig. 11 The maximum seeking process in the phase plane  $Q_1 - D_1$  with initial condition  $D_1(0) = 0.11 \text{ day}^{-1}$

## Conclusion

In this paper an algorithm for ES control of a cascade of two anaerobic bioreactors has been investigated, using the dilution rate of the first bioreactor  $D_1$  as control action and the biogas flow rate of the first bioreactor  $Q_1$  as measured output to be maximized. The control algorithm has been tested on a nonlinear model with Monod type of kinetics for both bioreactors. This algorithm is model free and is much easier to be realised practically than the model-based algorithms as in [2, 4].

In conclusion, our theoretical analysis and simulation studies show that it is possible to optimize the steady-state operation of a cascade of two anaerobic bioreactors with the ES control law (35) applied on the first BR in presence of variations of the influent waste concentration  $S_{0i}$ , maximizing the total biogas production and with sensible decrease of the general output depollution parameter  $COD_2$ . Obviously, the results obtained are only guidelines for practitioners of ES controller and they should adjust the values of the tuning parameters  $\omega_h$ ,  $\omega$ ,  $a$  and  $k$ .

## Notations

AD	–	anaerobic digestion
BR <sub><i>j</i></sub>	–	bioreactor “ <i>j</i> ” ( <i>j</i> = 1,2)
ES	–	extremum seeking
$COD_j$	–	chemical oxygen demand (integral parameter for pollution/depollution) in BR <sub><i>j</i></sub> ( <i>j</i> = 1, 2), gO.dm <sup>-3</sup>
$D_j$	–	dilution rate for BR <sub><i>j</i></sub> ( <i>j</i> = 1,2), day <sup>-1</sup>
$X_{ij}$	–	concentration of microorganisms for population “ <i>i</i> ” in bioreactor “ <i>j</i> ”, ( <i>i, j</i> = 1, 2), g.dm <sup>-3</sup>
$\mu_{ij}$	–	specific growth rate of population “ <i>i</i> ” in bioreactor “ <i>j</i> ”, day <sup>-1</sup>
$S_{ij}$	–	concentration of substrate “ <i>i</i> ”, in bioreactor “ <i>j</i> ”, ( <i>i</i> = 0, 1, 2, <i>j</i> = 1, 2), g.dm <sup>-3</sup>
$Q_j$	–	biogas flow rate from bioreactor “ <i>j</i> ”, dm <sup>3</sup> .day <sup>-1</sup>
<i>t</i>	–	time, day
d/dt	–	first time derivative

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