D. S. JONES, M. J. PLANK, B. D. SLEEMAN
DIFFERENTIAL EQUATIONS AND MATHEMATICAL BIOLOGY

The conjoining of mathematics and biology has brought about significant advances in both areas, with mathematics providing a tool for modelling and understanding biological phenomena and biology stimulating developments in the theory of nonlinear differential equations. The continued application of mathematics to biology holds great promise and in fact may be the applied mathematics of the 21st century.

Differential Equations and Mathematical Biology provides a detailed treatment of both ordinary and partial differential equations, techniques for their solution, and their use in a variety of biological applications. The presentation includes the fundamental techniques of nonlinear differential equations, bifurcation theory, and the impact of chaos on discrete time biological modelling. The authors provide generous coverage of numerical techniques and address a range of important applications, including heart physiology, nerve pulse transmission, chemical reactions, tumour growth, and epidemics.

This book is the ideal vehicle for introducing the challenges of biology to mathematicians and likewise delivering key mathematical tools to biologists. Carefully designed for such multiple purposes, it serves equally well as a professional reference and as a text for coursework in differential equations, in biological modelling, or in differential equation models of biology for life science students.

New to the Second Edition is:
- A section on spiral waves
- Recent developments in tumor biology
- More on the numerical solution of differential equations and numerical bifurcation analysis
- MATLAB® files available for download online
- Many additional examples and exercises.

Table of Contents
1. Introduction
   1.1 Population growth 1
   1.2 Administration of drugs 4
   1.3 Cell division 9
   1.4 Differential equations with separable variables 11
   1.5 Equations of homogeneous type 14
   1.6 Linear differential equations of the first order 16
   1.7 Numerical solution of first-order equations 19
   1.8 Symbolic computation in MATLAB 24
   1.9 Notes 27
2. Linear Ordinary Differential Equations with Constant Coefficients 33
   2.1 Introduction 33
   2.2 First-order linear differential equations 35
   2.3 Linear equations of the second order 36
   2.4 Finding the complementary function 37
   2.5 Determining a particular integral 41
   2.6 Forced oscillations 50
   2.7 Differential equations of order \( n \) 52
   2.8 Uniqueness 55

3. Systems of Linear Ordinary Differential Equations 61
   3.1 First-order systems of equations with constant coefficients 61
   3.2 Replacement of one differential equation by a system 64
   3.3 The general system 66
   3.4 The fundamental system 68
   3.5 Matrix notation 72
   3.6 Initial and boundary value problems 77
   3.7 Solving the inhomogeneous differential equation 82
   3.8 Numerical solution of linear boundary value problems 84

4. Modelling Biological Phenomena 91
   4.1 Introduction 91
   4.2 Heartbeat 91
   4.3 Nerve impulse transmission 94
   4.4 Chemical reactions 100
   4.5 Predator–prey models 106

5. First-Order Systems of Ordinary Differential Equations 115
   5.1 Existence and uniqueness 115
   5.2 Epidemics 118
   5.3 The phase plane and the Jacobian matrix 119
   5.4 Local stability 121
   5.5 Stability 128
   5.6 Limit cycles 133
   5.7 Forced oscillations 139
   5.8 Numerical solution of systems of equations 143
   5.9 Symbolic computation on first-order systems of equations and higher-order equations 147
   5.10 Numerical solution of nonlinear boundary value problems 149
   5.11 Appendix: existence theory 153

6. Mathematics of Heart Physiology 163
   6.1 The local model 163
   6.2 The threshold effect 166
   6.3 The phase plane analysis and the heartbeat model 168
   6.4 Physiological considerations of the heartbeat cycle 171
   6.5 A model of the cardiac pacemaker 173
   6.6 Notes 175

7. Mathematics of Nerve Impulse Transmission 177
   7.1 Excitability and repetitive firing 185
   7.2 Travelling waves 187
   7.3 Qualitative behavior of travelling waves 190
   7.4 Piecewise linear model 194
8. Chemical Reactions
   8.1 Wavefronts for the Belousov-Zhabotinskii reaction
   8.2 Phase plane analysis of Fisher’s equation
   8.3 Qualitative behavior in the general case
   8.4 Spiral waves and $\lambda - \omega$ systems
   8.5 Notes

9. Predator and Prey
   9.1 Catching fish
   9.2 The effect of fishing
   9.3 The Volterra-Lotka model

10. Partial Differential Equations
    10.1 Characteristics for equations of the first order
    10.2 Another view of characteristics
    10.3 Linear partial differential equations of the second order
    10.4 Elliptic partial differential equations
    10.5 Parabolic partial differential equations
    10.6 Hyperbolic partial differential equations
    10.7 The wave equation
    10.8 Typical problems for the hyperbolic equation
    10.9 The Euler-Darboux equation
    10.10 Visualization of solutions

11. Evolutionary Equations
    11.1 The heat equation
    11.2 Separation of variables
    11.3 Simple evolutionary equations
    11.4 Comparison theorems
    11.5 Notes

12. Problems of Diffusion
    12.1 Diffusion through membranes
    12.2 Energy and energy estimates
    12.3 Global behavior of nerve impulse transmissions
    12.4 Global behavior in chemical reactions
    12.5 Turing diffusion driven instability and pattern formation
    12.6 Finite pattern forming domains
    12.7 Notes

13. Bifurcation and Chaos
    13.1 Bifurcation
    13.2 Bifurcation of a limit cycle
    13.3 Discrete bifurcation and period-doubling
    13.4 Chaos
    13.5 Stability of limit cycles
    13.6 The Poincaré plane
    13.7 Averaging

14. Numerical Bifurcation Analysis
    14.1 Fixed points and stability
    14.2 Path-following and bifurcation analysis
    14.3 Following stable limit cycles
    14.4 Bifurcation in discrete systems
    14.5 Strange attractors and chaos
14.6 Stability analysis of partial differential equations 384
14.7 Notes 385

15. Growth of Tumors 389
15.1 Introduction 389
15.2 Mathematical model I of tumor growth 392
15.3 Spherical tumor growth based on model I 395
15.4 Stability of tumor growth based on model I 399
15.5 Mathematical model II of tumor growth 401
15.6 Spherical tumor growth based on model II 404
15.7 Stability of tumor growth based on model II 406
15.8 Notes 407

16. Epidemics 411
16.1 The Kermack-McKendrick model 411
16.2 Vaccination 413
16.3 An incubation model 414
16.4 Spreading in space 418

Answers to Selected Exercises 427

Index 439