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Journal of Biomechanics 32 (1999) 689–694

JOURNAL OF BIOMECHANICS

About weight factors in the non-linear objective functions used for solving indeterminate problems in biomechanics

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Received in final form 7 February 1999

Abstract

A lot of non-linear objective criteria are applied for solving the indeterminate problems formulated for different biomechanical models — most of them can be covered by the expression $\sum c_i|\vec{\mathbf{F}}_i|^n$. It might be noted, however, that most of the suggested criteria are not applicable if considerable antagonistic co-contractions exist. This could be an effect of treating the agonistic muscles and their respective antagonists in one and the same manner in the objective function. Using a completely inverse approach (the muscle forces are supposed to be known quantities) and a simple 1DOF model (actuated by three agonistic muscles and one corresponding antagonist) it has been shown which values of the weight factors c_i may predict different levels of muscle forces from the two antagonistic groups. Three hypothetical border variants for magnitudes of the muscle forces are considered (flexor muscles are only active, extensor muscles are only active, considerable co-contraction of flexors and extensors exists). The main conclusions are: the signs of c_i at agonistic muscles have to be opposite to the c_i signs at their antagonists; the signs of the weight factors depend on the direction of the net external joint moment; the closer c_i to zero, the bigger force will be predicted in the ith muscle.

Keywords: Indeterminate problem; Objective function; Antagonistic co-contraction; Elbow joint

1. Introduction

Different objective functions have been applied to solve the indeterminate problems formulated for different biomechanical models (details on definitions and basic concepts could be found in Herzog and Binding, 1994). Most of the criteria can be covered by the expression $\sum c_i|\vec{\mathbf{F}}_i|^n$, where $|\vec{\mathbf{F}}_i|$ is the module of the ith muscle force and c_i is a weight factor (Tsirakos et al., 1997). Many authors connect c_i with muscle physiological cross-sectional area (PCSA) or maximal muscle force (F_{max}). For example: $n = 2$, $c_i = (1/\mathrm{PCSA}_i)^2$ (An et al., 1984); $n = 3$, $c_i = (1/\mathrm{PCSA}_i)^3$ (Crowninshield and Brand, 1981); $n = 3$, $c_i = 1$ (Pedersen et al., 1987; Prilutsky and Gregor, 1997); $n = \alpha$, $c_i = (1/\mathrm{PCSA}_i)^\alpha$ (Herzog and Binding, 1993). As it could be noticed, the weight factors of all muscles have the same signs and similar magnitudes by nature. Notwithstanding that Hughes and Chaffin (1988) proved that coactivation of antagonistic muscles could not be predicted, if the first partial derivatives of the objective function with respect to the design variables are positive, objective functions with different signs of c_i are rarely found. Cholewichki and McGill (1994) use a specific function $\sum M_i(1 - g_i)^2$, where M_i is the moment of the ith muscle (M_i may have positive as well as negative values) and g_i is an individual muscle gain. However, it is stated in their paper that no physiological basis exists for choosing such a type of an objective function. Herzog and Binding (1993), using a planar 2DOF model, conclude that a co-contraction of pairs of one-joint antagonistic muscles is not possible, although they recognize that such co-contractions are observed experimentally under certain conditions. They predict simultaneous activity in a pair of two-joint antagonists. This is a consequence of the chosen situation in the joints, however — the direction of the net external moment in the proximal joint is opposite to that in the distal joint. The antagonistic activities predicted in complex, multi-degree of freedom models (Pedersen et al., 1987; Brand et al., 1986) could be related to the fact that the muscle moment is a vector.

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0021-9290/99/$ - see front matter
PII: S0021-9290(99)00037-8

When a muscle contracts it not only causes the necessary moment (due to muscle primary function), but also other moments (due to its secondary functions). For neutralization of the parasite moments, other muscles have to be active. The same is valid for the mechanico-chemical optimization criterion used in Patriarco et al. (1981). Patriarco's criterion is linear, all weight factors are positive, but it is used for force predictions during human gait, where different agonistic and antagonistic muscle groups are active during different step phases. Here, the importance of the joint torques is mentioned, but a suggestion about their eventual inclusion in an optimization function has not been made.

Another approach for identifying suitable objective functions, i.e. for searching different weight factors, is related to the contractile condition of the muscle (Herzog, 1987; Happee and Van der Helm, 1995; Kaufman et al., 1991b). The weight factors used in Herzog (1987) are $c_i = (1/M_i)^2$. Here M_i is the maximum instantaneous joint moment caused by the action of the ith muscle and $M_i = r_i F_i$ (r_i is the lever arm of the ith muscle). Using Hill's force–velocity relationship, F_i is expressed by the PCSA, the activation of the muscle, its instantaneous length and instantaneous rate of change in length. All c_i are positive (independent of r_i signs) and the ith weight factor is inverse proportional to the ith muscle lever arm squared. It is stated that the metabolic cost of movement will be low using such an objective function. The mathematical formulation and experiments (thought and real) have been devoted to one-degree of freedom models and antagonistic muscle activities have not been taken into account. Investigating fast goal directed movements, using a shoulder model with 95 muscle elements, Happee and Van der Helm (1995) propose an objective function with $c_i = v_i(1/F_{i\,\max})^2$, where v_i is the muscle volume. They relate the proportion $(F_i/F_{i\,\max})$ with calcium ion concentration, respectively, with muscular energy consumption. The weight factors are time dependent, but they are always positive. In spite of the explicitly written statement "antagonistic activity will never be predicted by any criterion of a form like equation …", a discussion is presented about antagonistic activity of thoracoscapularis muscle. Kaufman et al. (1991a, b) used a modified length–tension relationship accounting for muscle architecture, but only to define the upper limits of the muscle forces. An antagonistic activity is not obvious from the figures presented. The "minimum-fatigue criterion" (Dul et al., 1984a, b) is often used (Prilytsky and Gregor, 1997). The endurance time of the muscle, $T_i = \text{constant}(F_i)^n$, is included in the optimization procedure. The constant is a function of $F_{i\,\max}$ and of the percentage of slow-twitch fibers, n is a negative non-integer number. Surprisingly, in contrast to the conclusions made by many authors (Brand et al., 1986; Herzog, 1987; Raikova, 1996) about the sensitivity of load sharing to the muscle moment arms, the opposite has been stated here. This contradiction follows from the way the analytical solution is derived. First, the proportion between the muscle forces is obtained and then the moment equation is solved.

It is not expected that some of the formulations of optimization problems for biomechanical models will be ideal. Probably, the optimization criteria are task dependent (Nieminen et al., 1995). However, specific peculiarities observed by different authors by means of natural, thought, and numerical experiments have to be taken into account in the process of designing new objective criteria.

The purpose of this paper is to investigate which sets of weight factors and powers in the objective function $\sum c_i|\mathbf{F}_i|^n$, could predict different levels of activities in muscles from two antagonistic groups. The approach is opposite to those used up to now — supposing that muscle forces are known, the corresponding weight factors are calculated. Using a simple 1DOF model, three hypothetical border variants for magnitudes of the muscle forces driving the joint are considered (flexor muscles are only active, extensor muscles are only active, a considerable co-contraction of flexors and extensors exists).

2. Methods

2.1. Mathematical evidence

Let us consider a simple model shown in Fig. 1. Three muscle forces ($\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$) from a synergistic group perform the counterclockwise rotation of the body about the fixed point O. The action of a muscle belonging to the respective antagonistic group is presented by the force $\vec{F}_4$. An external moment, $\vec{M}_{\text{ext}}$, due to the gravity forces, the inertial forces and the external loading, is applied to the body. Supposing that both $\vec{M}_{\text{ext}}$ and the lever arms of the muscle forces, d_i, are known, the moment equation with respect to the centre O is written in scalar terms as

$$M = d_{\text{ext}}|\vec{M}_{\text{ext}}|, \tag{1}$$

where $M = \sum_{i=1}^{4} d_i \mathbf{F}_i$ is the sum of the muscle moments, $\mathbf{F}_i$ is the magnitude of the ith muscle force, $d_{\text{ext}} = (+1)$ if $\vec{M}_{\text{ext}}$ has a clockwise direction and $d_{\text{ext}} = (-1)$ if $\vec{M}_{\text{ext}}$ has a counterclockwise direction.

Let us consider the indeterminate problem defined by Eq. (1) and the inequality constraints $\mathbf{F}_i > 0$ (i = 1, 2, 3, 4). Let us suppose that the distribution of the $\mathbf{M}_{\text{ext}}$ among the individual muscles is performed on the basis of some optimization criterion having the form of $Z = \sum_{i=1}^{4} c_i \mathbf{F}_i^n$ ($n > 1$) and c_i are unknown weight factors. The main question is: which c_i could be used? The necessary conditions for existence of an extremum (minimum, maximum or saddle point) of Z, when the equality constraints are

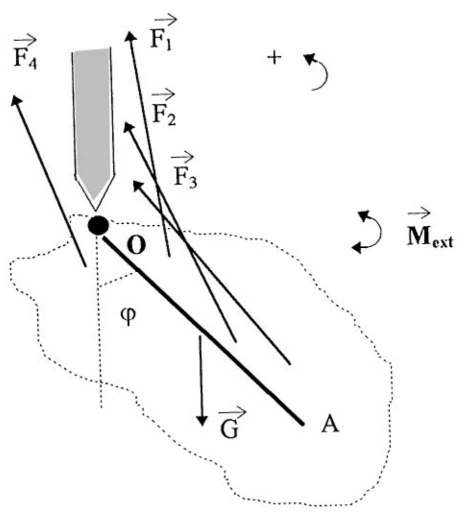

Fig. 1. One-degree of freedom model actuated by three synergistic muscle ($\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$) and one antagonist of theirs ($\vec{F}_4$). $\vec{G}$ is the gravity force, $\vec{M}_{ext}$ is the external moment in the joint.

fulfilled, lead to the requirement that all first partial derivatives of the Lagrange function are zero (see for details in Raikova, 1996). Consecutively, expressions for muscle forces could be obtained:

$$nc_i\mathbf{F}_i^{n-1} - \lambda d_i = 0 \Rightarrow \mathbf{F}_i = \sqrt[n-1]{\frac{\lambda d_i}{nc_i}} \quad (i = 1, 2, 3, 4), \qquad (2)$$

where λ is the Lagrange multiplier. Since the directions of the moments of the muscles belonging to the two antagonistic groups are different, d_i are positive numbers for $i = 1, 2, 3$, but $d_4 < 0$. From the requirements $\mathbf{F}_i > 0$ there follows that $\lambda/c_i > 0$ for $i = 1, 2, 3$ and $\lambda/c_4 < 0$. Hence, the signs of the weight coefficients of the agonistic muscles should be opposite to the sign of the weight coefficient of the antagonistic muscle. If $\lambda > 0$, c_1, c_2 and c_3 will be positive and c_4 will be negative. If $\lambda < 0$, c_1, c_2 and c_3 will be negative and c_4 will be positive.

Let us imagine that the individual muscle forces are known. Then it follows from (2)

$$\left(\frac{\mathbf{F}_i}{\mathbf{F}_4}\right)^{n-1} = \frac{c_4}{c_i}\frac{d_i}{d_4}, \quad (i = 1, 2, 3). \qquad (3)$$

So, the conclusion about the signs of c_i becomes clearer. By using Eq. (3), the proportion between the magnitudes of the weight coefficients could be calculated if the proportion between the magnitudes of the muscle forces is known. The inverse proportion between c_i and the respective muscle force could be noticed. The bigger the $\mathbf{F}_i$ is with respect to $\mathbf{F}_4$, the closer c_i is to zero. Hence, a bigger muscle force is associated with a smaller value of the respective weight factor.

Since min(const. $f(x)$) = const.min $f(x)$ if const > 0 and min(const. $f(x)$) = const.max $f(x)$ if const < 0, it is sufficient to calculate c_i for only one value of c_4, following the expression:

$$c_i = c_4\left(\frac{d_i}{d_4}\frac{\mathbf{F}_4^{n-1}}{\mathbf{F}_i^{n-1}}\right) \quad (i = 1, 2, 3). \qquad (4)$$

It may be concluded from Eqs. (2) and (4) that if the weight coefficient c_4 is multiplied by a constant, all other c_i, as well as λ and Z, will be multiplied by the same constant. The same results will be obtained using different values of the muscle forces if the proportions $\mathbf{F}_4/\mathbf{F}_i$ remain the same. The value of the moment M, however, will be different.

3. Results and discussion

3.1. Numerical experiments

An illustration for elbow flexion–extension is suggested using the model shown in Fig. 1. Let the joint be driven by three flexors: BIC (m.biceps brachii $-\vec{F}_1$), BRA (m.brachialis $-\vec{F}_2$), BRD (m.brachioradialis $-\vec{F}_3$) and one extensor: TRI (m.triceps brachii $-\vec{F}_4$). The data used is taken from Lemay and Crago (1996) and is summarized in Table 1. Three hypothetical border variants will be considered.

Variant 1: It is supposed that the flexor muscles are predominantly active and the extensor is nearly silent. For example, let the forces developed by the flexors are equal to 50% of their maximal values and $\mathbf{F}_4 = 0.1$[N] (see Table 2). By using Eq. (4), the weight coefficients c_i ($i = 1, 2, 3$) could be calculated for different values of c_4 (in Table 2 only the calculations with $c_4 = -100$ are submitted). It is evident from Table 2, that the increase of the power n causes the decrease of c_1, c_2 and c_3. The cost of the objective function, Z_{ext}, decreases, too. If other values of d_i are used, the results change, but not to a significant degree.

Variant 2. Let us consider a hypothetical mirror variant — the flexor muscles are nearly silent and the force of the m.TRI is about 50% of the maximal available force of this muscle (see Table 2). Since the sum of the moments of all muscle forces M is negative, $d_{ext} = (-1)$. Hence, the external moment in the joint has a counterclockwise

Table 1
Data used for muscle moment arms (d_i), physiological cross-sectional area ($PCSA_i$) and maximal muscle force ($F_{\max i}$) calculated using the value 30 [N] cm^{-2} (Dul et al., 1984b). φ is the joint angle

	$PCSA_i$ [m^2]	d_i[m] $\varphi = 30°$	d_i[m] $\varphi = 60°$	$F_{\max i}$ [N]
F_1(BIC)	4.6×10^{-4}	0.0204	0.0340	138
F_2(BRA)	7×10^{-4}	0.0151	0.0198	210
F_3(BRD)	1.33×10^{-4}	0.0244	0.0376	39.9
F_4(TRI)	18.8×10^{-4}	-0.0250	-0.0225	564

Table 2
Computed weight coefficients (c_i), Lagrange multiplier (λ) and value of the objective function (Z_{ext}) for different values of the muscle forces (F_i) and power (n) of the objective function. M is the sum of the muscle moments

	n	c_1	c_2	c_3	c_4	λ	Z_{ext}
Variant 1	2	2.190×10^{-1}	8.381×10^{-2}	8.376×10^{-1}	-100	8.889×10^{2}	2.299×10^{3}
$\varphi=60^\circ$							
$F_1=69$[N]	3	3.174×10^{-4}	7.982×10^{-5}	4.199×10^{-3}	-100	1.333×10^{2}	2.299×10^{2}
$F_2=105$ [N]							
$F_3=19.95$[N]	4	4.600×10^{-7}	7.602×10^{-8}	2.105×10^{-5}	-100	1.778×10^{1}	2.299×10^{1}
$F_4=0.1$[N]							
$M=5.173$[N.m]	5	6.667×10^{-10}	7.240×10^{-11}	1.055×10^{-7}	-100	2.222×10^{0}	2.299×10^{0}
Variant 2	2	-1.151×10^{2}	-5.678×10^{1}	-2.752×10^{2}	0.1	-2.256×10^{3}	7.940×10^{3}
$\varphi=30^\circ$							
$F_1=0.2$[N]	3	-1.622×10^{5}	-5.337×10^{4}	-7.762×10^{5}	0.1	-9.543×10^{5}	2.239×10^{6}
$F_2=0.3$[N]							
$F_3=0.1$[N]	4	-2.287×10^{8}	-5.017×10^{7}	-2.189×10^{9}	0.1	-3.588×10^{8}	6.314×10^{8}
$F_4=282$[N]							
$M=-7.039$[N.m]	5	-3.225×10^{11}	-4.716×10^{10}	-6.172×10^{12}	0.1	-1.265×10^{11}	1.781×10^{11}
Variant 3	2	-3.335×10^{-3}	-1.622×10^{-3}	-1.380×10^{-2}	0.001	-2.256×10^{1}	4.027×10^{1}
$\varphi=30^0$							
$F_1=69$[N]	3	-1.363×10^{-2}	-4.357×10^{-3}	-1.950×10^{-1}	0.001	-9.543×10^{3}	1.136×10^{4}
$F_2=105$[N]							
$F_3=19.95$[N]	4	-5.570×10^{-2}	-1.170×10^{-2}	-2.757×10^{0}	0.001	-3.588×10^{6}	3.203×10^{6}
$F_4=282$[N]							
$M=-3.570$[N.m]	5	-2.277×10^{-1}	-3.143×10^{-2}	-3.896×10^{1}	0.001	-1.265×10^{9}	9.031×10^{8}

direction. For illustration, the calculations are performed with $c_4 = 0.1$. It could be noticed that all other c_i are negative, their absolute values are much bigger than c_4 and increase rapidly when the power n increases. In contrast to the first variant, Z_{ext} increases with n and λ is negative.

Variant 3: This is a borderline hypothetical case — all muscles develop 50% of their maximal forces. Since the maximal force of m.TRI is the biggest, M is negative and the external joint moment has a counterclockwise direction. Here the absolute values of all weight coefficients have similar orders, since all forces are essentially different from zero. In contrast to the previous variants, these values are not much influenced by the power n.

The type of extremum is numerically investigated for the three variants (see Fig. 2). One of the muscle forces has a fixed value, two are changed near the optimal point. The fourth muscle force is calculated from the moment equation (if this force is negative, there is no solution — see Fig. 2b). Then the value of the objective function, Z, is computed and compared with the Z_{ext}. It could be noticed that the objective function reaches its local maximum for Variants 2 and 3 and its local minimum for Variant 1. If all c_i for Variants 2 and 3 are multiplied by (-1) the type of extremum will be minimum too.

Independent of some authors' statement that only a co-contraction of two-joint muscles could be predicted (Herzog and Binding, 1993, 1994), which implies that the hypothetical Variant 3 would not be observed in a real situation, the existence of one-joint antagonistic muscles should not be ignored, even if their forces are nearly zero (*Variants 1* and *2*). Certainly depending on the direction of the external moment, either the forces of the flexor muscles or the forces of the extensor muscles can be set to zero in the Eq. (1). However, muscles that are nonactive, for a certain period of time or posture, are also controlled by the human brain. Hence, if an objective criterion has a claim to reflect somehow the motor control of the human brain, the nonactive muscles must be also included in this criterion.

4. Conclusion

Different mathematical approaches are applied in the motor control study — neural network models (Nussbaum and Chaffin, 1997; Koike and Kawato 1994, 1995), stiffness and impedance control (Hogan et al., 1987), equilibrium point (trajectory) hypothesis (Latash, 1992; Gotlieb, 1994; Flash, 1987), maximum smoothness theory of coordination (minimization of jerk) (Hogan and Flash, 1987; Hagan et al., 1987; Krylow and Rymer, 1997). The present paper is addressed to the optimization techniques, where muscle forces are included in the objective functions as design variables.

The main findings from the performed calculations are: if an objective criterion having the form $\sum c_i|\vec{\mathbf{F}}_i|^n$ is used to solve an indeterminate problem and antagonistic

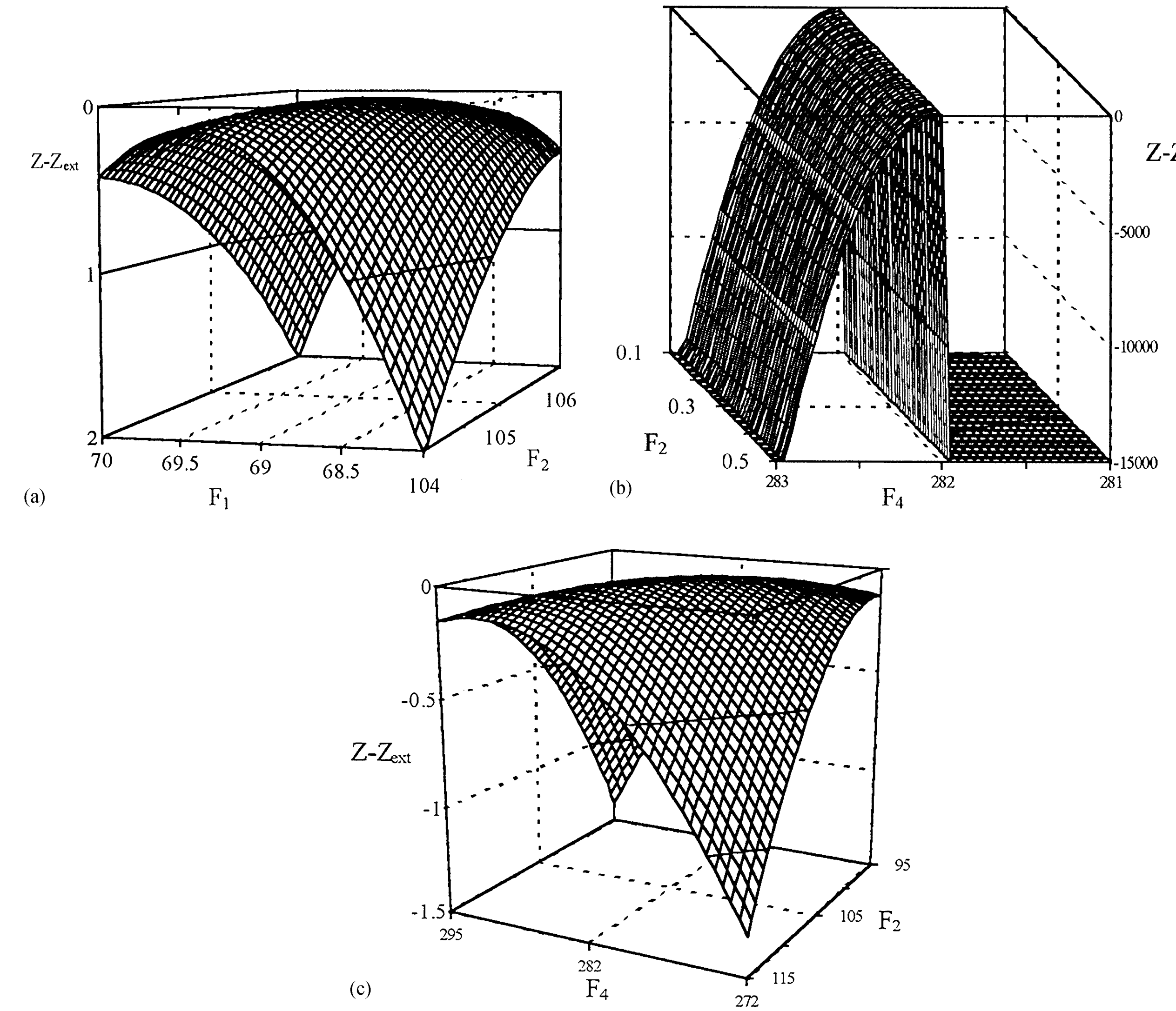

Fig. 2. Investigation of the type of extremum for the three variants listed in Table 2 for $n = 2$ (muscle forces are given in $[N]$). Fig 2(a) Variant 1, $F_1 \in [68, 70]$, $F_2 \in [104, 106]$, $F_4 = 0.1$, F_3 is calculated from the moment equation, $Z = 0.219F_1^2 + 0.08381F_2^2 - 0.8376F_3^2 - 100F_4^2$, $Z_{ext} = 2299$; Fig. 2(c) Variant 2, $F_2 \in [0.1, 0.5]$, $F_4 \in [281, 283]$, $F_3 = 0.1$, F_1 is calculated from the moment equation, $Z = -115.1F_1^2 - 56.78F_2^2 - 275.2F_3^2 + 0.1F_4^2$, $Z_{ext} = 7940$, there are no possible solutions for $F_4 > 282$; Fig 2(c) Variant 3, $F_2 \in [95, 115]$, $F_4 \in [272, 292]$, $F_3 = 19.95$, F_1 is calculated from the moment equation, $Z = -0.00335F_1^2 - 0.001622F_2^2 - 0.0138F_3^2 + 0.001F_4^2$, $Z_{ext} = 40.27$.

muscles are included in the model too, the weight factors c_i of the muscles from the two different groups (agonist – antagonistic) must have different signs, and these signs are connected with the direction of the net external moment in the joint. The absolute value of the weight factor is connected with the value of the muscle force in an inverse proportion — more force will be predicted for a muscle whose $|c_i|$ is closer to zero. The conclusions about the signs and the values of the weight factors in the objective function $\sum c_i|\vec{\mathbf{F}}_i|^n$, drawn by using the so proposed inverse approach, confirm the previous investigations (Raikova, 1996; Raikova and Dimitrov, 1996). The sets of the possible values of c_i have been investigated using analytical solutions of the optimization tasks for 1DOF (Raikova, 1996) and 3DOF (Raikova and Dimitrov, 1996) planar models. The relationship between the net joint moments and the signs and values of the weight factors has been explicitly observed.

The general question about the physiological interpretation and analytical expressions of these weight factors remains. They could be functions of different morphological characteristics of the muscles (PCSA, moment arms, maximal muscle forces, volume), of the current and the previous states of the muscles (change of the length, fatigue, contraction velocity), of the current loading of the limbs (joint moments and joint reactions or contact forces), of the voluntary brain control and so on. Further numerical, thought and natural experiments might help in designing new, more complex objective criteria providing possibilities for an adequate description of the human limb driving system.

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