

# Investigation of the Peculiarities of Two-joint Muscles using a 3 DOF Model of the Human Upper Limb in the Sagittal Plane: an Optimization Approach

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The purpose of this paper is an investigation of the peculiarities of biarticular muscles by means of modelling and analytical solution of the indeterminate problem. The basic model includes 10 muscle elements performing flexio/extension in the shoulder, elbow and wrist. Four of them are biarticular muscles. Two modifications of the model with only monoarticular muscles are developed. The indeterminate problem is solved analytically using the objective criterion  $\sum c_i F_i^2$ , where  $F_i$  is the module of the  $i$ -th muscle force and  $c_i$  is a weight coefficient. The predicted muscle forces, joint reactions and moments are compared in-between the basic model and its two modifications for different joint angles, external loading and weight coefficients. The main conclusions are: it is impossible to formulate strict advantages of the biarticular muscles under quasistatistical conditions, their peculiarities depend on limb position, external loading and neural control; in general, monoarticular muscles are more powerful than biarticular ones; the biarticular muscles fine tune muscle coordination, their control is more precise and graceful; the presence of biarticular muscles leads to an increase of the joint reactions and moments, thus stabilizing the limb.

*Keywords:* Modelling; Optimization; Indeterminate problem; Upper limb; Biarticular muscles

## 1. INTRODUCTION

During the earliest investigations of the human and animal limbs the existence of two-joint (2-joint, biarticular) and poliarticular (multiarticular) muscles has attracted scientific attention [1, 2]. The uniqueness of the muscles crossing more than one joint is related to the opinion that no motor

act exists, that could not be performed by a set of one-joint (1-joint, monoarticular, uniarticular, single) muscles [3]. The specific actions of these muscles are systematized in Van Ingen Schenau [3–5]. Hogan *et al.* [6] are of opinion that: “single joint muscles are sufficient to generate the necessary torques and poliarticular muscles would appear to be an excess, a redundancy in the

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system". The paradoxical behavior of the two-joint muscles of the lower limb is mentioned during cycling (Lombard's Paradox). The following question naturally arises [7]: since fully antagonistical biarticular muscles of the lower limb exist (for example m.hamstrings performing simultaneous hip extension and knee flexion and m.rectus femoris performing simultaneous hip flexion and knee extension) which of these muscles will be active when the motions of the two joints are with opposite directions (for example during simultaneous hip and knee flexio) and how the human brain controls muscle activities.

Different approaches for investigating the characteristics of the two-joint muscles have been used. Most of the investigations consider human lower limb: in statics, during controlling an external force [8]; in dynamics - during jumping and jogging [9–11], walking and running [5, 12] and standing [13]. Fewer papers consider upper human limb. Different combinations of flexio/extensio [14–16] or flexio (extensio) and pronatio (supinatio) of the elbow [17, 18] are experimentally investigated. Often the attention is paid to work, energy and power [9, 19–21]. The contribution of the muscles to the angular acceleration of the joint is considered by Zajac and Gordon [13] aiming to find the specific actions of uni- and multi-articular muscles. The coordination between one- and two-joint muscles in [17, 18] is investigated recording surface electromyography (EMG) activity from eight muscles driving elbow joint. Gielen *et al.* [15] use both surface and intramuscular EMG signals. Hogan *et al.* [6] use the "endpoint stiffness" parameter to prove the advantages of the poliarticular muscles. They state that the presence of poliarticular muscles has a dramatic influence on the total end-point stiffness of the limb. De Lussanet and Alexander [22] look for a relationship between hand end-point velocity and time delay between one- and two-joint muscle activities for fast movements. Smeets [16] investigates two-joint muscles from the point of view of sensory and motor accuracy. An overview on the possibility for different type of control of mono- and bi-articular

muscles is presented in Van Ingen Shenau [4], but it remains unclear whether this difference is related to differences in morphological and mechanical characteristics of these muscles or in neural mechanism for their control.

There exist many hypotheses about advantages of biarticular muscles, but less is mentioned about their disadvantages. The difficulties in investigation and generalization of the actions of the biarticular muscles as well as in a comparison between one- and two-joint muscles are obvious. According to Zajac and Gordon [23] biarticular muscles are "enigmatic", because their actions may vary depending on motor task, limb position, kinematic characteristics and so on. One of the advantages of the two-joint muscles mentioned by many authors [24–26] is the fact that they act in many cases almost isometrically. This situation is observed when the motion in the first joint causes shortening of the two-joint muscle (for example rectus femoris during hip flexion) and in the second joint - lengthening (rectus femoris during knee flexio). In such a way the muscle works well in terms of force-length and force-velocity relationships and the contraction velocity may decrease [3]. This peculiarity is also related to the positive and negative work done by a two-joint muscle and with the dissipation of the energy [15]. It is stated that with biarticular muscles the negative work in a joint can be avoided, thus enhancing the movement coordination. Another feature of the two-joint muscles is the so-called "coupling of the joint movements" [3–6]. This means that controlling only one actuator (a multiarticular muscle), rotations in more than one joint arise. This is why the biarticular muscles are called also "activatable mechanical links" between the joints [25]. According to Zajac [10] and Zajac and Pandy [27], uni-articular muscles deliver most of the power and energy and bi-articular muscles fine-tune muscle coordination. Another hypothesis about the specific roles of these muscles is connected with the statement that two-joint muscles allow an independent control of the position and force of the end point [3, 28].

Probably this also serves to achieve a smooth and fine movement control. Prilutsky and Gregor [8], however, state that one-joint muscles may also play an important part in controlling external force. Much attention is paid to the functions of the two-joint muscles from energetic point of view, because of these muscles abilities to “transport the power” [5] or to “transfer mechanical energy” [9]. A few authors explain what these terms mean from the point of view of mechanics and mathematics. Prilutsky and Zatsiorsky [9] clearly define their understanding of “transfer of mechanical energy” and show that the direction of this transfer is different during different phases of the step cycle. In Prilutsky *et al.* [20] it is shown (using the term “mechanical energy expenditure”) that during locomotion the presence of two-joint muscles could lead to saving mechanical energy.

In most of the papers devoted to the applications of optimization methods for solving the force-sharing problem (see for review in Tsirakos *et al.* [29] and for definitions and basic concepts in Herzog and Binding [30]), no special attention is paid to the role of the two-joint muscles, despite of their presence in the models [31–34]. It is rarely mentioned how the two-joint muscles are modelled and how they are included in the optimization procedure. Here one can apply either of the following two approaches: each two-joint muscle can be represented by one force that has different moment arms with respect to the rotation centers of the two joints which the muscle crosses and the weight factor of the muscle force in the objective function is constant; or the moment equations could be solved consecutively from distal to proximal links and the predicted force of a two-joint muscle can be added as a known quantity in the equilibrium equations of the next link. In Prilutsky and Gregor [8] a static motor task (pushing on the ground or pulling a strap) is simulated in the sagittal plane. The model of the lower extremity consists of four rigid elements and three frictionless hinge joints (hip, knee, ankle). Nine muscles drive the joints; three of them are two-joint ones. The two-joint muscles have

different arms with respect to the two joint centers. Three frequently used optimization criteria are applied and the force sharing is compared. The weight coefficients of the muscle forces in the objective functions are constants. The following conclusions are drawn about the behavior of a biarticular muscle (see also [35]): it produces more force when it could contribute to the resultant moments at both joints (*i.e.*, when the direction of the moment of the muscle force is opposite to the directions of the net external moments in the two joints); its force is always zero when it acts as an antagonist in both joints (*i.e.*, when the direction of the moment of the muscle force coincides with the direction of the external moments of the two joints); its activity is lower when it acts as an antagonist in one joint and agonist in the other one (*i.e.*, for intermediate cases between the first two). The good idea proposed in [20] for a substitution of the two-joint muscles with two equivalent one-joint muscles is not applied here and only a comparison between the three optimization criteria is made. Herzog and Binding [24] replace a two-joint muscle with two energetically equivalent uniarticular muscles so that the geometry of the models with and without two-joint muscles remains the same. In order to compare the behavior of the two models, in such a way that the value of the objective function be the same, the objective function is taken to be  $\Sigma(F_i^2/V_i)$ , where  $F_i$  and  $V_i$  are the  $i$ -th muscle force and volume. The mass of a two-joint muscle is distributed between two monoarticular muscles. Changing the moments in the two joints, using an analytical solution of the optimization problem, the authors investigate the value of the objective function and possibilities for muscle co-contractions. For the model with only monoarticular muscles the predicted non-zero muscle forces strictly follow the direction of the moments in the distal and proximal joints. If these moments have identical directions, then muscles performing flexio (or extensio) have non-zero forces, otherwise the flexors of one joint and the extensors of the other joint are active. Regions exist where this regularity

is violated for the model with two-joint muscles - co-contractions of one- and two- joint muscles are observed. It is shown (comparing the values of the objective function) that regions exist where the model with biarticular muscles is more cost effective than one with monoarticular muscles only, but there exist as well other regions where the opposite case takes place. The volumes of these regions depend both on the chosen signs and values of the joint moments and on the way the two-joint muscles are split into one-joint muscles. The main conclusion of the authors is: in some cases models containing 2-joint muscles allow for a more effective performance of a task, whereas in other situations, an energetically equivalent model containing just 1-joint muscles would be more effective. The results of this paper, however, are limited because of many model simplifications concerning the geometry of the model and the lever arms and volumes of the muscles.

The aim of the present paper is to investigate the peculiarities of the two-joint muscles by comparing a model containing biarticular muscles with two modifications of it where these muscles are replaced by monoarticular ones. A complex enough planar model of the human upper limb with 3 DOF is used for this purpose. It includes ten muscle elements performing flexio and extensio in the shoulder, elbow and wrist, four of them biarticular ones. Two modifications of the model with only one-joint muscles are constructed. The first one assumes that biarticular muscles act only in one of the two joints they cross. In the second modification, each of the two-joint muscles is represented by two its geometrically equivalent (with respect to the lever arm and angle between muscle force vector and longitudinal axes of the upper arm segments) monoarticular muscles. The optimization problem for distribution of the efforts among the muscles is solved analytically using Lagrange multipliers method and the objective function  $\sum c_i F_i^2$ , where  $F_i$  is the module of the  $i$ -th muscle force and  $c_i$  is an unknown weight factor. The behavior of the three models is compared observing the predicted muscle forces,

joint reactions and moments for different configurations of the models and different weight coefficients  $c_i$ .

## 2. METHODS

### 2.1. Description of the Model with Biarticular Muscles (MBM) and its Two Modifications (MOD1 and MOD2) for which the Two-joint Muscles are Replaced by Monoarticular Ones

The basic model of the upper limb (**MBM**) is shown in Figure 1. It is in the sagittal plane and has three degrees of freedom - flexio/extensio in the shoulder ( $\varphi_1$ ), elbow ( $\varphi_2$ ) and wrist ( $\varphi_3$ ) joints (see Fig. 1a). The joints are frictionless. The presence of joint capsules and ligaments is ignored. Ten actuators (nine muscles presented by 10 forces) drive the model links - see Figure 1b and Table I. It is assumed that they formally describe the actions of the primary flexors and extensors of the human upper limb. Four biarticular muscles are modelled - BIC, TRI, EDI and FCR (for abbreviations see Tab. I). The line of action of each muscle unit is modelled by different numbers of "segments" [36] - parts of straight lines. For simplicity, the muscle arms are assumed independent from the joint angles. Each of the muscle force is described by its inventory vector:  $\mathbf{Inv}_i = (\mathbf{ds}_i, \mathbf{de}_i, \mathbf{dw}_i, \alpha_{i,1}, \alpha_{i,2}, \alpha_{i,3})$ , where  $\mathbf{ds}_i$ ,  $\mathbf{de}_i$  and  $\mathbf{dw}_i$  are the  $i$ -th muscle moment arms with respect to the rotation centers of the shoulder, elbow and wrist joints, respectively,  $\alpha_{i,1}$ ,  $\alpha_{i,2}$  and  $\alpha_{i,3}$  are the angles between  $i$ -th muscle force and  $O_1X_1$ ,  $O_2Y_2$ ,  $O_3Y_3$  axes, respectively (see [37] for details, Figure 1a for definition of the local coordinate systems and Figure 1c for explanation of the components of the inventory vector). The used values of the components of  $\mathbf{Inv}_i$  for all muscle elements are given in Table I. Lack of relation between a muscle and a joint is denoted by "Ø". For such cases the moment arm, cosine and sine of the angle are set

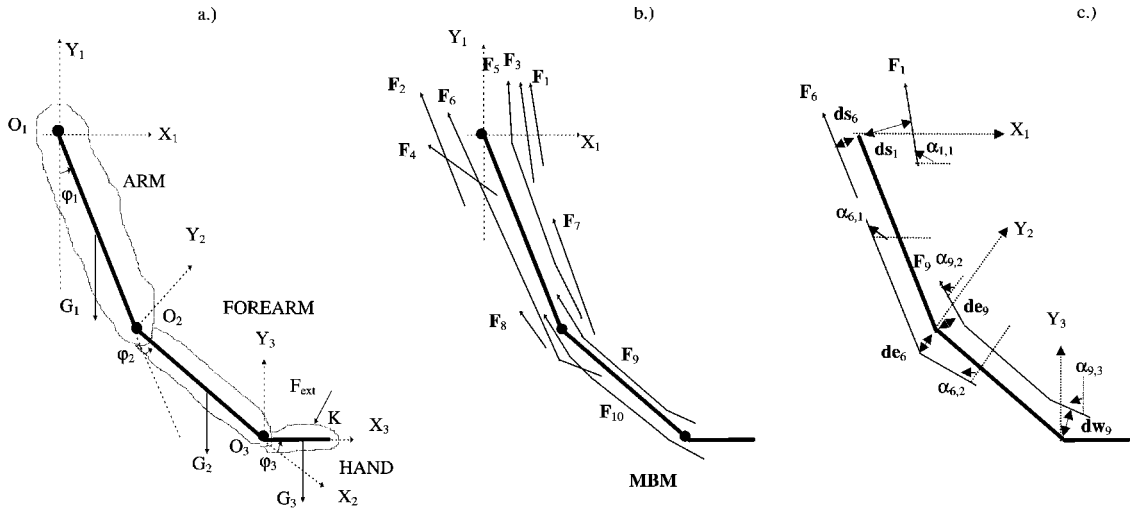


FIGURE 1 The basic model of the human upper limb in the sagittal plane (**MBM**). Figure 1a: used coordinate systems.  $O_1$ ,  $O_2$  and  $O_3$  - rotation centers of the shoulder, elbow and wrist joints respectively;  $\varphi_1$ ,  $\varphi_2$  and  $\varphi_3$  - flexion-extension angles in the shoulder, elbow and wrist joints;  $G_1$ ,  $G_2$  and  $G_3$  - gravity forces of the arm, forearm and hand;  $O_iX_iY_i$  orthogonal coordinate systems;  $F_{ext}$  - external force applied to the hand. The values of the link lengths used for computations are: arm length,  $|O_1O_2| = 32[\text{cm}]$ , forearm length,  $|O_2O_3| = 30[\text{cm}]$  and hand length,  $|O_3K| = 19[\text{cm}]$ . It is supposed that the application points of the gravity forces,  $G_i$ , are in the middles of the respective links. The used values for the arm weight is  $G_1 = 1.6G$  and for the hand weight is  $G_3 = 0.4G$ , where  $G = G_2$  is the forearm weight ( $\approx 10\text{N}$ ). Figure 1b: modeled muscle forces  $F_i$  (see Tab. I for subscripts  $i$ ).  $F_5$ ,  $F_6$ ,  $F_9$  and  $F_{10}$  represent the actions of the biarticular muscles BIC, TRI, FCR and EDI. Figure 1c. Explanation of the components of the inventory vector of the muscles:  $ds_i$ ,  $de_i$  and  $dw_i$  are the  $i$ -th muscle moment arms with respect to the rotation centers of the shoulder, elbow and wrist joints respectively;  $\alpha_{i,1}$ ,  $\alpha_{i,2}$ ,  $\alpha_{i,3}$  are the angles between the  $i$ -th modelled muscle force and the axis  $O_1X_1$ ,  $O_2Y_2$  and  $O_3Y_3$  respectively.

TABLE I Muscles included in the basic model **MBM**, forces with which they are modelled and values of the parameters of the respective inventory vectors.  $ds_i$ ,  $de_i$  and  $dw_i$  are the lever arms of the  $i$ -th muscle force with respect to the axes of rotation in the shoulder, elbow and wrist joints respectively. (+) counterclockwise direction of the muscle moment, (-) clockwise direction of the muscle moment;  $\alpha_{i,1}$ ,  $\alpha_{i,2}$ ,  $\alpha_{i,3}$  are the angles between  $i$ -th muscle force and  $O_1X_1$ ,  $O_2Y_2$ ,  $O_3Y_3$  axes respectively (see Fig. 1). If a muscle does not act in a particular joint, the respective arm and angle are denoted as  $\emptyset$ .

Muscle	Abbreviation	Modelled muscle force	Inventory vector of the muscle					
			$ds_i$	$de_i$	$dw_i$	$\alpha_{i,1}$	$\alpha_{i,2}$	$\alpha_{i,3}$
m.deltoideus (pars clavicularis)	DEL p.cl.	$F_1$	5.2	$\emptyset$	$\emptyset$	$103^\circ$	$\emptyset$	$\emptyset$
m.deltoideus (pars spinata)	DEL p.sp.	$F_2$	-6.2	$\emptyset$	$\emptyset$	$120^\circ$	$\emptyset$	$\emptyset$
m.coracobrachialis	COR	$F_3$	3.9	$\emptyset$	$\emptyset$	$100^\circ$	$\emptyset$	$\emptyset$
m.teres major	TMJ	$F_4$	-5.3	$\emptyset$	$\emptyset$	$125^\circ$	$\emptyset$	$\emptyset$
m.biceps brachii	BIC	$F_5$	3.6	3.0	$\emptyset$	$95^\circ$	$67^\circ$	$\emptyset$
m.triceps brachii	TRI	$F_6$	-4.0	-2.8	$\emptyset$	$130^\circ$	$100^\circ$	$\emptyset$
m.brachialis	BRA	$F_7$	$\emptyset$	3.4	$\emptyset$	$\emptyset$	$55^\circ$	$\emptyset$
m.anconeus	ANC	$F_8$	$\emptyset$	-1.4	$\emptyset$	$\emptyset$	$80^\circ$	$\emptyset$
m.flexor carpi radialis	FCR	$F_9$	$\emptyset$	0.9	1.1	$\emptyset$	$73^\circ$	$70^\circ$
m.extensor digitorum	EDI	$F_{10}$	$\emptyset$	-1.1	-0.94	$\emptyset$	$75^\circ$	$60^\circ$

to zero during subsequent calculations. The inventory vector of one monoarticular muscle comprises two components only - an arm and an angle. Each of the two-joint muscles is represented by one force, but the line of action of this force is modelled by more than one segment (see Fig. 1b). The inventory vector of a biarticular muscle has four non-zero components:  $\mathbf{ds}_i$ ,  $\mathbf{de}_i$ ,  $\alpha_{i,1}$  and  $\alpha_{i,2}$  for the muscles acting simultaneously in the shoulder and elbow joints;  $\mathbf{de}_i$ ,  $\mathbf{dw}_i$ ,  $\alpha_{i,2}$  and  $\alpha_{i,3}$  for the muscles acting simultaneously in the elbow and wrist joints (see Fig. 1c). Two modifications of this basic model are developed where the two-joint muscles are replaced by monoarticular ones. They are shown in Figure 2. For **MOD1**, muscle BIC (represented by force  $F_5$ ) acts as an elbow flexor only, m.TRI ( $F_6$ ) acts as a shoulder extensor only, m.FCR ( $F_9$ ) acts as a wrist flexor only, m.EDI ( $F_{10}$ ) acts as a wrist extensor only. For **MOD2**, each of the two-joint muscles is replaced by two mono-articular muscles, equivalent to the corresponding biarticular muscle from a geometrical point of view only (with respect to the arms and angles  $\alpha_{i,j}$ ), *i.e.*, four forces are added to the

previous **MOD1**. The force  $F_{11}$  represents the action of m.BIC in the shoulder joint (see Fig. 2, **MOD2**). The forces  $F_{12}$ ,  $F_{13}$  and  $F_{14}$  represent the actions of the muscles TRI, FCR and EDI in the elbow joint. So, **MOD1** and **MOD2** differ from the basic model **MBM** only in the inventory vectors of the two-joint muscles. For example, the inventory vector of m.BIC is  $\mathbf{Inv}_5 = (3.6, 3.0, \emptyset, 95^\circ, 67^\circ, \emptyset)$  for **MBM** and  $\mathbf{Inv}_5 = (\emptyset, 3.0, \emptyset, \emptyset, 67^\circ, \emptyset)$  for **MOD1**. For **MOD2** two inventory vectors for this muscle are defined:  $\mathbf{Inv}_5 = (\emptyset, 3.0, \emptyset, \emptyset, 67^\circ, \emptyset)$  and  $\mathbf{Inv}_{11} = (3.6, \emptyset, \emptyset, 95^\circ, \emptyset, \emptyset)$ .

## 2.2. Equations of Equilibrium and Optimization Procedure

The computational algorithm is described in details in [37]. Briefly, the sequence of steps is: (1) composition of the free-body diagrams of each of the three bodies (arm, forearm and hand); (2) derivation of three algebraic equations for moment equilibrium of the joints and six algebraic equations for force equilibrium in components (using projections of all forces on the axes of the

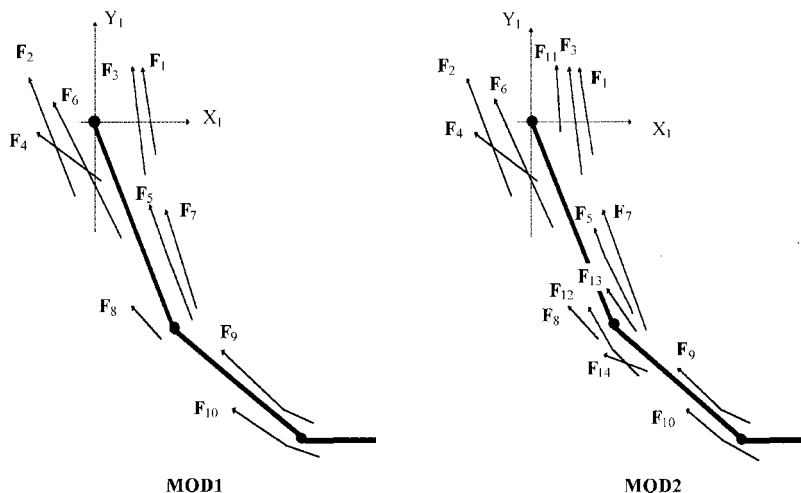


FIGURE 2 Two modifications of the basic model **MBM** (see Fig. 1b) containing only monoarticular muscles. **MOD1** - m.BIC acts as elbow flexor only ( $F_5$ ), m.TRI acts as shoulder extensor only ( $F_6$ ), m.FCR acts as wrist flexor only ( $F_9$ ), m.EDI acts as wrist extensor only ( $F_{10}$ ); **MOD2** - each of the two-joint muscles is represented by two monoarticular ones: the action of m.BIC in the shoulder is presented by the force  $F_{11}$  and in the elbow - by  $F_5$ ; the action of m.TRI in the shoulder is presented by the force  $F_6$  and in the elbow - by  $F_{12}$ ; the action of m.FCR in the elbow is presented by the force  $F_{13}$  and in the wrist - by  $F_9$ ; the action of m.EDI in the elbow is presented by the force  $F_{14}$  and in the wrist - by  $F_{10}$ .

coordinate systems  $O_iX_iY_i$  ( $i=1,2,3$ ) - see Figure 1a and Appendix); (3) building of a system of three algebraic equations about the unknown muscle forces by expression of the components of the joint reactions as functions of the muscle forces and consecutive substitutions of these components in the moment equations. An optimization approach is used for solving the obtained three equations about the unknown muscle forces. The number of the latter is 10 for **MBM** and **MOD1**, whereas for **MOD2** it is 14. This approach is based on the Lagrange multipliers method [38]. The objective function is assumed to have a complex form:  $Z(F_i) = \sum c_i \cdot F_i^2$ , where  $F_i$  is the module of the  $i$ -th muscle force and  $c_i$  is a weight factor of the  $i$ -th muscle force. The weight factors  $c_i$  may be either positive or negative [39,40], provided that the obtained system of three algebraic equations and the inequality constraints  $F_i \geq 0$  are met. Previous investigations [39–41] have shown that the weight factors of the muscles from two anatomically antagonistic groups have opposite signs and that the closer the weight factor to zero, the more force is predicted in the respective muscle. The physiological interpretation and experimental validation of the objective criterion are beyond the scope of the present paper. It is supposed here that  $c_i$ -s somehow reflect the human brain control on the muscle activities and that these weight coefficients may be changed arbitrarily provided that all constraints are met. From this point of view, a set of strictly positive muscle forces that respect the equality constraints and the conditions for existence of an extremum of the objective function  $\sum c_i F_i^2$  for some set of  $c_i$ -s could be viewed as one among many possible equilibrium points. In fact, using different  $c_i$ -s, different possible motor control strategies are considered. The question is not which of these strategies is really used. The aim is comparing the behavior of the three models **MBM**, **MOD1** and **MOD2** under strictly identical conditions to extract peculiarities of the two-joint muscles.

### 2.3. Scheme of the Numerical Experiments

The input parameters for the numerical experiments are: the lengths and the weights of the arm, forearm and hand, the applied external force ( $F_{ext}$ ) to the hand, the values of the joint angles ( $\varphi_1, \varphi_2, \varphi_3$ ) and the inventory vectors of the muscles ( $\mathbf{Inv}_i$ ). The calculated parameters that are monitored are: the predicted muscle forces ( $F_i$ ); the sum of these forces ( $S_{tot}$ ); the reactions in the shoulder ( $R_{sh}$ ), elbow ( $R_{el}$ ) and wrist ( $R_{wr}$ ) joints; the total muscle moments in the shoulder ( $M_{sh}$ ), elbow ( $M_{el}$ ) and wrist ( $M_{wr}$ ) joints (see Appendix). During the numerical experiments only positive muscle forces are searched, since the muscles can only pull, but not push. If some of the muscle forces becomes negative it is said that there is no solution for the current parameters.

The computational program is implemented on a PC [37, 41]. Two types of numerical experiments were performed:

- (1) It is supposed that the control strategy is the same for **MBM**, **MOD1** and **MOD2**, *i.e.*, the weight factors  $c_i$  (suitably chosen so that a solution of the optimization problem exists) are the same for the basic model and for its two modifications. It is supposed that  $c_{11} = c_5$ ,  $c_{12} = c_6$ ,  $c_{13} = c_9$  and  $c_{14} = c_{10}$  for **MOD2** because  $F_5$  and  $F_{11}$  are the forces with which m.BIC acts in the elbow and shoulder respectively,  $F_6$  and  $F_{12}$  are the forces with which m.TRI acts in the shoulder and elbow respectively,  $F_9$  and  $F_{13}$  are the forces with which m.FCR acts in the wrist and elbow respectively,  $F_{10}$  and  $F_{14}$  are the forces with which m.EDI acts in the wrist and elbow respectively. So, it is assumed that the muscle as a whole is controlled by the brain, but individual motor units are not. Fixing suitably chosen  $c_i$ -s, the output parameters are traced out for different joint angles, with and without external force (*i.e.*,  $F_{ext} = 0$  or  $F_{ext} \neq 0$ ).
- (2) Fixing the same joint angles and external force for the three models, the influence of the

weight factors  $c_i$  on the output parameters is investigated. Usually  $c_i$  is changed iteratively, using suitable increment, identical for the three models. Decreasing the module of the  $c_i$  of a muscle, its predicted force increase [41]. So, the aim of these experiments was to see how the change of this force influences the other predicted muscle forces.

### 3. RESULTS AND DISCUSSION

The results fall into two completely different situations: (A) - all net external moments in the joints have clockwise direction and (B) - some of these moments are with counterclockwise direction. The reason for this is that the behaviour of a biarticular muscle when its moment counters (or resists) the net external moments in both joints is very different from the case when its moment direction coincides with the direction of one of the net external joint moment, but is opposite to the other net external joint moment [35]. It should be mentioned that the predicted muscle forces, joint reactions and moments in the present paper are overestimated during the computations naturally due to the following: the upper limits of the muscle forces are not included in the computational algorithm as inequality constraints since sub-maximal motor tasks are not considered; not all muscles of the upper human limb are included, the modeled muscles are rather muscle equivalents of a synergistic group; the role of the joint capsules and ligaments for the maintenance of the joint stability is not taken into account, but they can reduce joint reactions and moments.

#### 3.1. Cases when all Net External Moments in the Joints Have a Clockwise Direction

These cases are more often actually encountered because of the gravity force actions. Obviously the flexor muscles are primarily active here and the extensors are silent. Hence, the total muscle

moments in the joints  $M_{sh}$ ,  $M_{el}$  and  $M_{wr}$  are positive (the positive direction is chosen to be counterclockwise one, see also Appendix).

Removing the one-joint shoulder flexors COR and DELp.cl. from the model with biarticular muscles **MBM**, *i.e.*, assuming that only the biarticular m.BIC is able to perform shoulder flexion, it was nearly impossible to find a set of  $c_i$ -s such that there can be a positive solution. The net external moment in the shoulder joint is usually the biggest one because of the effort transmissions *via* joints. The force of m.BIC is predicted by the optimization procedure mainly to satisfy the moment equation with respect to the elbow joint center. Its predicted force however remains insufficient for satisfying the moment equation with respect to the shoulder joint center. Decreasing the weight coefficient of m.BIC so that its force to increase, the moment of this force with respect to the elbow joint center increases too and becomes to surpass the net external moment in the elbow. Then more forces are needed in the elbow extensors in exchange. However, such an antagonistic co-contraction is energetically inefficient. The force of a two-joint muscle is mainly predicted according to the lower moment of the two joints it spans, whereas monoarticular muscles have no such restriction and their forces may be arbitrary big. Hence, it could be concluded that one-joint muscles are more powerful than the biarticular ones.

Fixing the weight coefficients  $c_i$  identical for the three models, changing a joint angle within its physiologically anatomical range it was observed that the patterns of muscle force distributions do not differ much among the three models (see Figs. 3a, b and c). The differences are between  $\varphi_2$ 's regions where possible solutions exist.  $\varphi_2 \in [2^\circ, 117^\circ]$  for **MBM** (Fig. 3a),  $\varphi_2 \in [0^\circ, 117^\circ]$  for **MOD1** (Fig. 3b) and  $\varphi_2 \in [0^\circ, 59^\circ]$  for **MOD2** (Fig. 3c). The increase of the flexion angle in the elbow causes significant changes in the predicted forces of m.DELp.cl. and COR because of the change of the moments of the gravity forces and the consecutive transmission of the efforts to the



## INVESTIGATION OF THE PECULIARITIES OF TWO-JOINT MUSCLES

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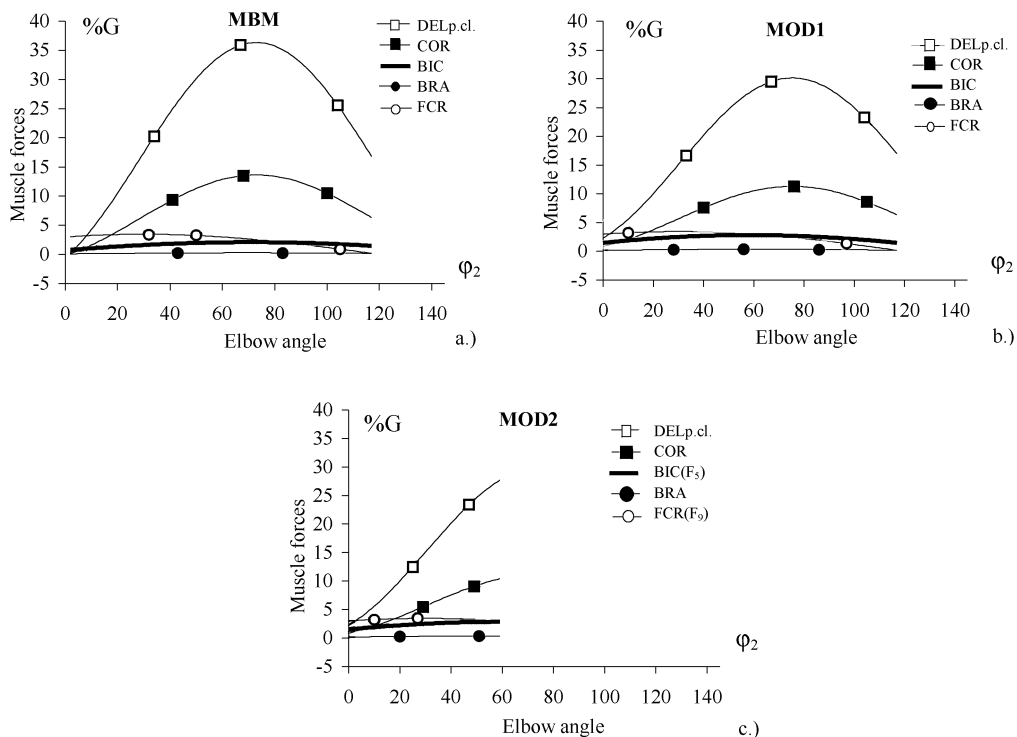


FIGURE 3 Comparison of the output parameters of **MBM**, **MOD1** and **MOD2**. The shoulder and wrist are flexed ( $\varphi_1 = 50^\circ$ ,  $\varphi_3 = 10^\circ$ ), the angle in the elbow,  $\varphi_2$ , is changed from  $0^\circ$  to  $140^\circ$ . The weight coefficients of the muscle forces have one and the same fixed values for the three models:  $c_1 = 0.0001$ ,  $c_3 = 0.0002$ ,  $c_5 = 0.01$ ,  $c_7 = 0.1$ ,  $c_9 = 0.7$ ,  $c_2 = c_4 = c_6 = c_8 = c_{10} = -1500$  (for **MOD2**  $c_{11} = c_5$ ,  $c_{12} = c_6$ ,  $c_{13} = c_9$ ,  $c_{14} = c_{10}$ ). All forces are calculated as relative to forearm weight  $G$ . Figures 3a, 3b and 3c - predicted muscle forces for **MBM**, **MOD1** and **MOD2** respectively (only essential different from zero forces are shown); Figure 3d - sums of all predicted muscle forces,  $S_{\text{Tot}}$ ; Figure 3e - sums of the predicted forces of the extensor muscles; Figure 3f - reactions ( $R_{\text{sh}}$ ) and moments ( $M_{\text{sh}}$ ) in the shoulder joint for the three models; Figure 3g - reactions ( $R_{\text{el}}$ ) and moments ( $M_{\text{el}}$ ) in the elbow joint for the three models; Figure 3h - reactions ( $R_{\text{wr}}$ ) and moments ( $M_{\text{wr}}$ ) in the wrist joint for the three models. The joint reactions are computed as percentage of forearm weight  $G$ , the length unit for moment calculations is centimeter.

shoulder joint (see also Figs. 3f and 3g). Independently of the increase of moment in the shoulder joint when  $\varphi_2$  nears to  $80^\circ$ , the force of the biarticular m.BIC (Fig. 3a) does not increase and this muscle does not help the monoarticular shoulder flexors. Its force is much lower than the forces of m.DELp.cl. and COR. The sum of all muscle forces is higher for the model with biarticular muscles for some values of the angle  $\varphi_2$  (Fig. 3d), but for other values this sum is the lowest. This ambiguity is not due to differences in the forces predicted in the antagonistic muscles (extensors in this case) - see Figure 3e. The sums of the predicted forces of the extensor muscles are very low (the main reason is, of course, the choice

of the weight coefficients at the extensors:  $c_2 = c_4 = c_6 = c_8 = c_{10} = c_{12} = c_{14} = -1500$ ) and do not differ significantly between the three models. The greatest differences between the three models were observed concerning the reactions and moments in the shoulder and elbow joints (Figs. 3f and 3g), but the reactions and moments in the wrist joint do not differ (Fig. 3h). It was observed that the quantities  $M_{\text{sh}}$ ,  $R_{\text{sh}}$  and  $R_{\text{el}}$  were nearly always the biggest for the model with biarticular muscles (**MBM**).

Many similar numerical experiments were performed using other joint angles and weight coefficients. The results show the following: the pattern of muscle force's distributions does not

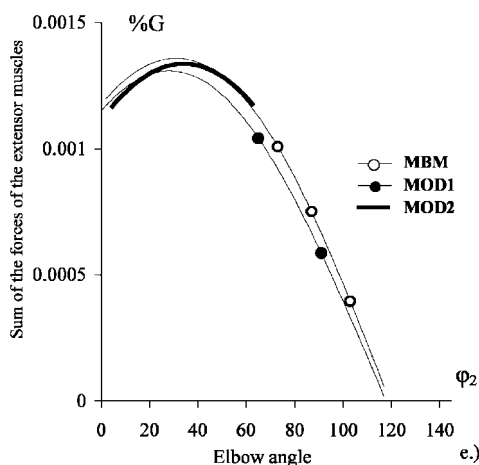
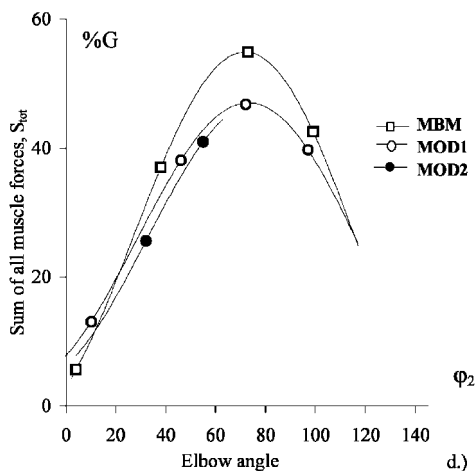


FIGURE 3 (Continued).

differ significantly between the three models, it could be changed by varying the muscle weight coefficients, *i.e.*, changing the control strategy, but not including (*i.e.*, **MBM**) or excluding (*i.e.*, **MOD1** and **MOD2**) the biarticular muscles; a stable conclusion about the advantages of some of the models can not be drawn, but it is likely that the presence of biarticular muscles offers more possibilities for different means of control on the driving system since possible solutions of the optimization task can be obtained for many more sets of  $c_i$  for **MBM** in comparison to the models without biarticular muscles; usually the regions of the joint angles where possible solutions exist were different for the three models and were larger for

**MBM**; the sum of all muscle forces may be higher or lower for **MBM** in comparison with the models containing only monoarticular muscles, *i.e.*, it could not be said which of the models is more effective from energetic point of view; nearly always  $M_{wr}$ ,  $R_{wr}$  and  $M_{el}$  coincided for the three models, whereas  $M_{sh}$ ,  $R_{sh}$  and  $R_{el}$  were the greatest for the model with biarticular muscles; almost always the force of m.BIC was the lowest for the model with biarticular muscles and the predicted forces of m.DELp.cl. and m.COR were greater than that of m.BIC; despite of setting  $c_5 = c_{11}$  and  $c_9 = c_{13}$  for **MOD2**, the values of the forces  $F_{11}$  and  $F_{13}$  were very different from these of  $F_5$  and  $F_9$ . Hence, the predicted force in a muscle depends not

INVESTIGATION OF THE PECULIARITIES OF TWO-JOINT MUSCLES

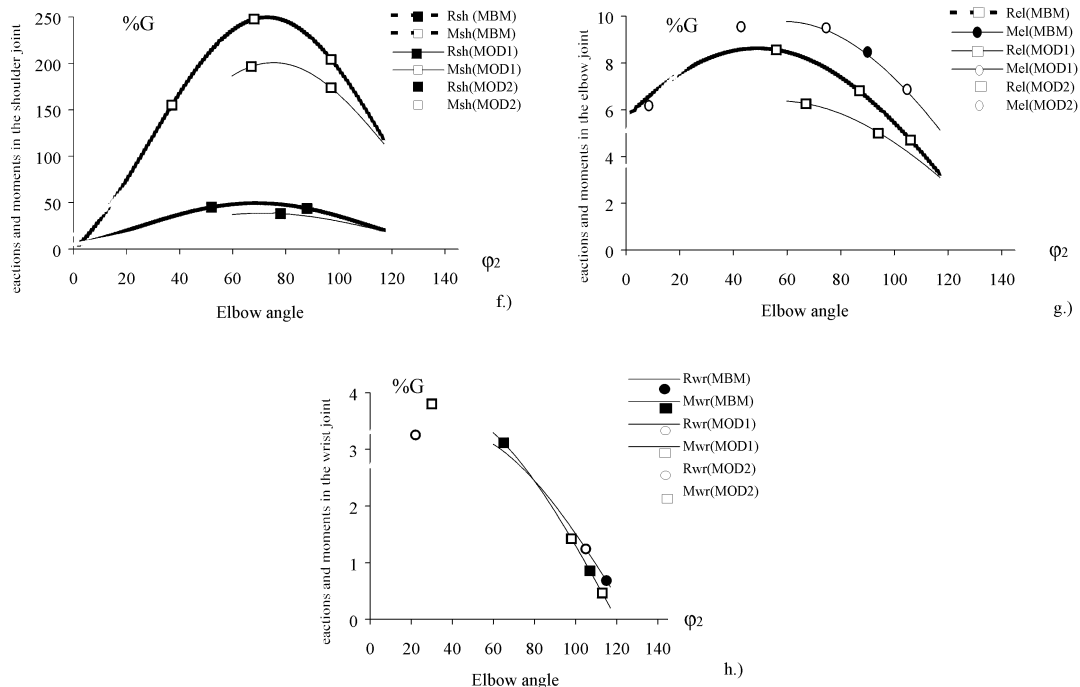


FIGURE 3 (Continued).

only on its weight factor in the objective function but also on the net external joint moment.

Fixing all input parameters equal for the three models and applying an increasing external force collinear to the gravity force of the hand, all output parameters increase proportionally to  $F_{ext}$ , but the maximal force that can be carried by the hand is different for the three models. No significant advantage was obtained of any of the models in this sense.

The purpose of the next numerical experiments was to investigate the influence of the muscle controls, *i.e.*, of the  $c_i$ -s, on the behaviour of the three models. Fixing the external force applied to the hand, as well as the values of the joint angles, one weight coefficient was changed iteratively, whereas the remaining  $c_i$ -s were fixed. In general, it was observed that the pattern of muscle force's distribution remained similar for the three models, but  $c_i$ 's regions where possible solutions exist were different for the three models. For example, possible solutions of the optimization task exist for  $c_5 \in [0, +\infty]$  for **MBM**, whereas  $c_5 \in [0, 0.5]$  for

**MOD1** and  $c_5 \in [0.24, 0.52]$  for **MOD2** (Fig. 4a). When  $c_5$  increases, *i.e.*, decreasing the force of m.BIC, the sums of the muscle forces (Fig. 4b), the forces of the muscles DELp.cl. and COR, the joint reactions and moments decrease, only the force of m.BRA increases and this of m.FCR does not change (Fig. 4a). Hence, for the current configuration ( $\varphi_1 = 30^\circ$ ,  $\varphi_2 = 25^\circ$ ,  $\varphi_3 = 20^\circ$ ) it is more effective if the biarticular m.BIC is not active. The weight coefficient of the biarticular muscles BIC and FCR can have big positive value for **MBM**, but not for **MOD1** and **MOD2**. The influence of the weight coefficient of m.FCR, however, is insignificant, since this is the only flexor of the wrist included in the model and there are no possibilities for distribution of the wrist moment among other muscles. Similarly to m.FCR, the change of the weight coefficient of m.EDI causes negligible changes in the output parameters. Changing, however, the weight coefficient of m.TRI from a big negative value to zero (*i.e.*, increasing its force modeling antagonistic co-contraction), nearly all output parameters of

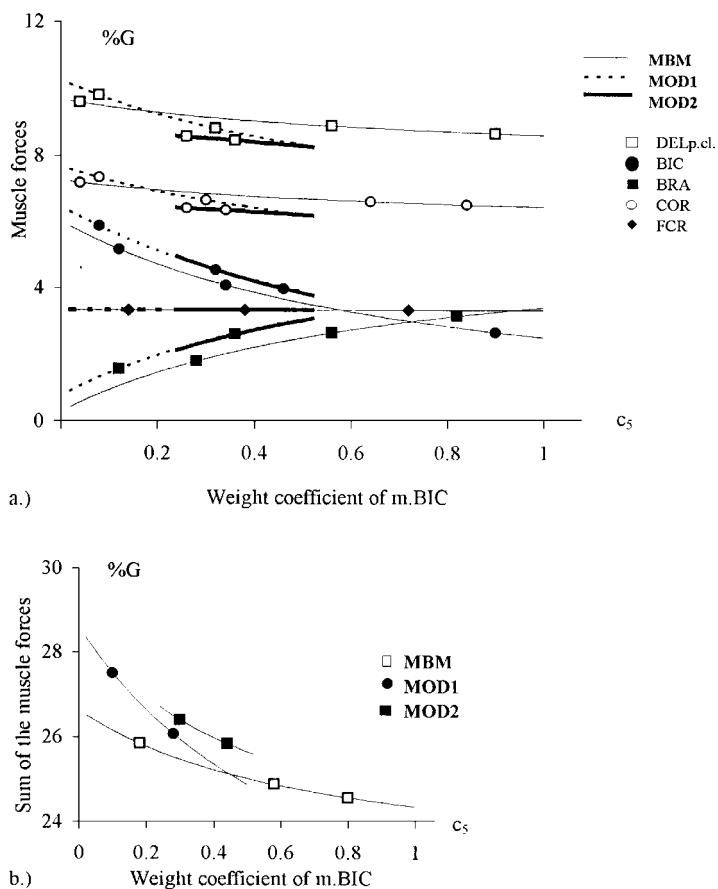


FIGURE 4 Muscle force distributions (Fig. 4a) and sum of all predicted muscle forces (Fig. 4b) for the three models when the weight coefficient of m.BIC,  $c_5$ , is changed from 0 to 1. The remaining weight coefficients are:  $c_1 = c_3 = 0.05$ ,  $c_7 = 0.9$ ,  $c_9 = 0.7$ ,  $c_2 = c_4 = c_6 = c_8 = c_{10} = -1500$  (for **MOD2**  $c_{11} = c_5$ ,  $c_{12} = c_6$ ,  $c_{13} = c_9$ ,  $c_{14} = c_{10}$ ). The joint angles are fixed as follows:  $\varphi_1 = 30^\circ$ ,  $\varphi_2 = 25^\circ$ ,  $\varphi_3 = 20^\circ$ . The forces are calculated as relative to forearm weight G.

**MBM** and **MOD2** increase, but they remain nearly constants for **MOD1** (see Fig. 5). Approaching  $c_6$  (remember that  $c_{12} = c_6$  for **MOD2**) to zero a little co-contraction of m.TRI in the shoulder joint is occurred (see TRI - **MOD1**, Fig. 5b and TRI ( $F_6$ ) - **MOD2**, Fig. 5c). The change of the force  $F_{12}$  that represents the action of m.TRI in the elbow joint is significant, however (Fig. 5c). As a result, because of an increase of the joint reaction in the elbow and consecutive addition of the moment of this reaction to the shoulder net external moment, the forces of the shoulder flexors (Fig. 5d),  $R_{sh}$  and  $M_{sh}$  increase rapidly. The same influence has the biarticular m.TRI (see Fig. 5a, **MBM**). Its co-

contraction influences both shoulder and elbow joints. In general, an antagonistic co-contraction of a biarticular muscle has a stronger reflection on the whole motor system than that of a mono-articular muscle (*i.e.*, TRI versus ANC).

### 3.2. Cases when the Net External Moments in the Joints Have Different Direction

Different configurations of the basic model and its two modifications are investigated in order to simulate situations for which the net external moments in the joints have different direction. Four such cases are shown in Figure 6 (Variants A, B, C

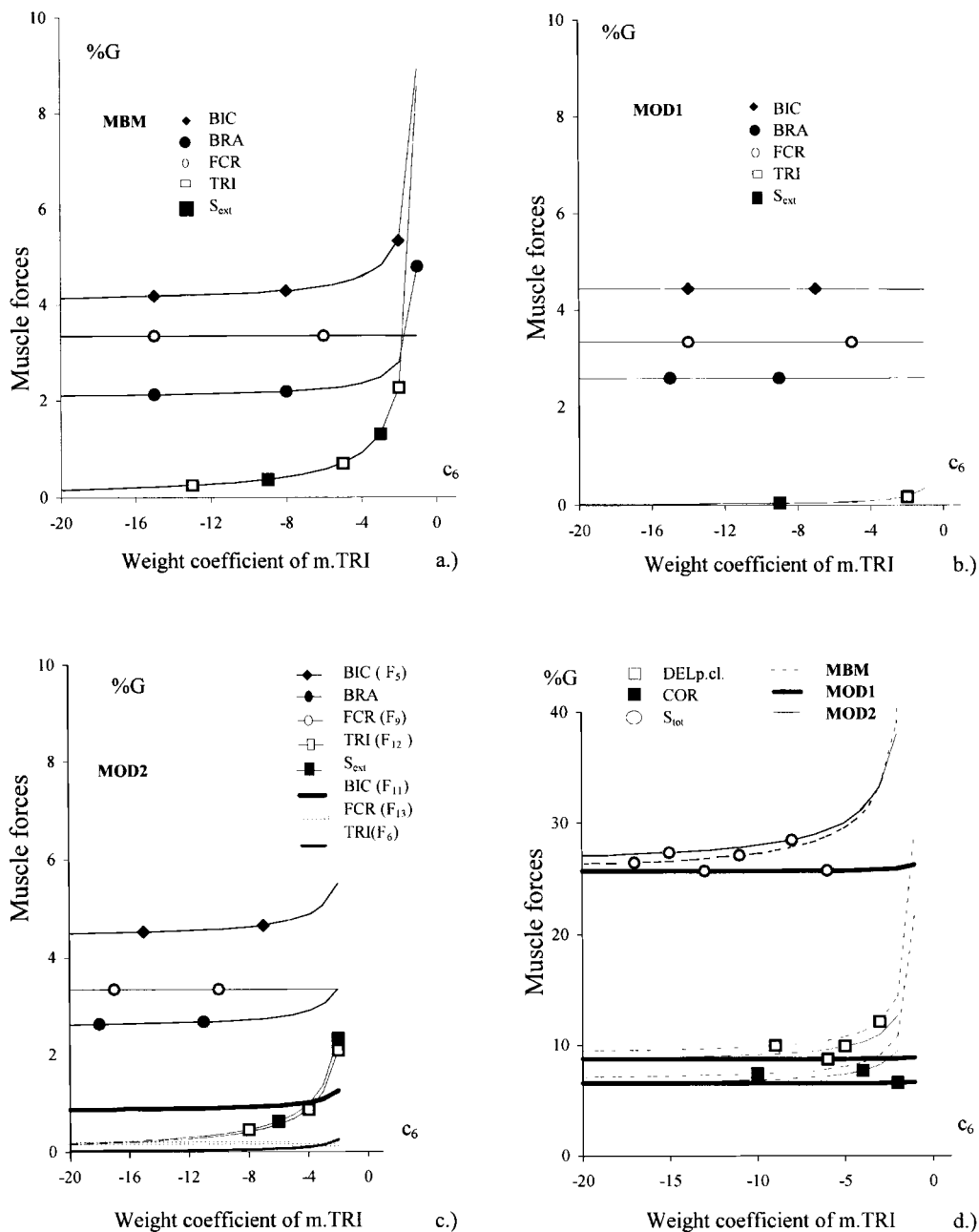


FIGURE 5 Output parameters of the three models when the weight coefficient of m.TRI,  $c_6$ , is changed from  $-20$  to  $0$ .  $F_{ext} = 0$ ,  $\varphi_1 = 30^\circ$ ,  $\varphi_2 = 25^\circ$ ,  $\varphi_3 = 20^\circ$ ,  $c_1 = 0.05$ ,  $c_3 = 0.05$ ,  $c_5 = 0.35$ ,  $c_7 = 0.9$ ,  $c_9 = 0.7$ ,  $c_2 = c_4 = c_8 = c_{10} = -1500$  (for **MOD2**  $c_{11} = c_5$ ,  $c_{12} = c_6$ ,  $c_{13} = c_9$ ,  $c_{14} = c_{10}$ ). The forces are calculated as relative to forearm weight  $G$ . Figure 5a. Predicted forces of the muscles BIC, BRA, FCR and TRI and the sum of the forces of all extensor muscles,  $S_{ext}$ , for the model with biarticular muscle **MBM**; Figure 5b. Predicted forces of the muscles BIC, BRA, FCR and TRI and the sum of the forces of all extensor muscles,  $S_{ext}$ , for the model with 10 monoarticular muscle **MOD1**; Figure 5c. Predicted forces of the muscles BIC ( $F_5$  and  $F_{11}$ ), BRA, FCR ( $F_9$  and  $F_{13}$ ) and TRI ( $F_6$  and  $F_{12}$ ) and the sum of the forces of all extensor muscles,  $S_{ext}$ , for the model with 14 monoarticular muscle **MOD2**; Figure 5d. Predicted forces of the muscles DEL.p.cl., COR and the sum of all predicted muscle forces,  $S_{tot}$ , compared in-between the three models.

**Variant A.**  $M_{sh}>0, M_{el}<0, M_{wr}<0$

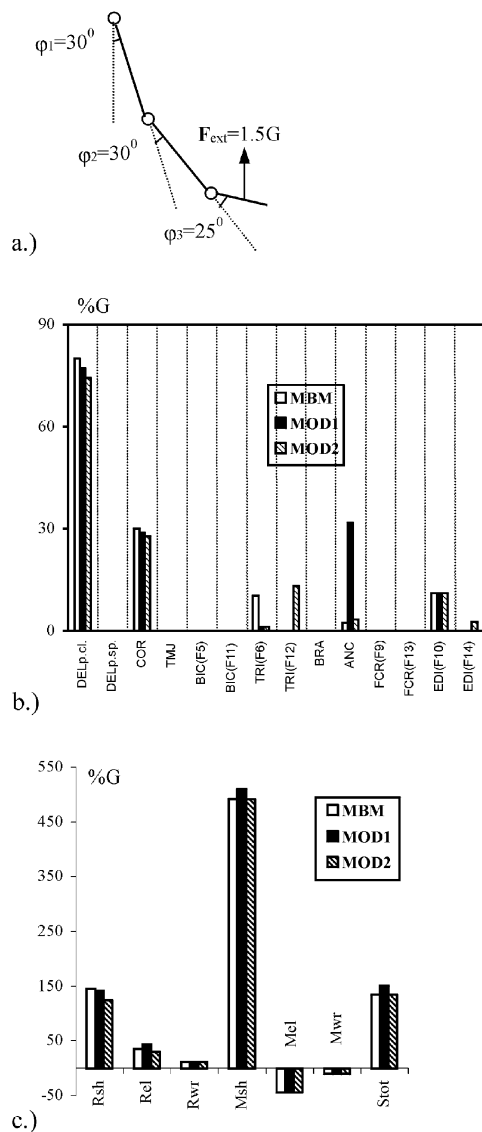


FIGURE 6 Four configurations of the models for which the net external moments in the shoulder, elbow and wrist joints have different directions.  $M_{sh}$ ,  $M_{el}$  and  $M_{wr}$  are the total muscle moment in the shoulder, elbow and wrist joints respectively. These moments are equal in magnitude but with opposite direction to the respective net external moment (for explanation see in the text). (a.) configuration of the models - joint angles  $\varphi_i$  and applied to the hand external force  $F_{ext}$  (G is forearm weight); (b.) predicted muscle forces compared in between the three models; (c.) joint reactions ( $R_{sh}$ ,  $R_{el}$ ,  $R_{wr}$ ) and moments ( $M_{sh}$ ,  $M_{el}$ ,  $M_{wr}$ ) compared in between the three models. The muscle forces and joint reactions are computed as percentage of forearm weight G, the length unit for moment calculations is centimeter. Variant A: the external force is with direction opposite to the gravity force of the hand, the weight coefficients are:  $c_1=0.0001, c_2=-250, c_3=0.0002, c_4=-250, c_5=250, c_6=-0.005, c_7=250, c_8=-0.01, c_9=250, c_{10}=-0.01$  (for **MOD2**  $c_{11}=c_5, c_{12}=c_6, c_{13}=c_9, c_{14}=c_{10}$ ); Variant B: the external force is with direction opposite to the gravity force of the hand, the weight coefficients are:  $c_1=0.001, c_2=-1500, c_3=0.002, c_4=-1500, c_5=0.5, c_6=-1500, c_7=0.5, c_8=-1500, c_9=1500, c_{10}=-0.5$  (for **MOD2**  $c_{11}=c_5, c_{12}=c_6, c_{13}=c_9, c_{14}=c_{10}$ ); Variant C: the external force is with the same direction as the gravity force of the hand, the weight coefficients are:  $c_1=0.00001, c_2=-1500, c_3=0.00002, c_4=-1500, c_5=1500, c_6=-0.001, c_7=1500, c_8=-0.01, c_9=0.9, c_{10}=-1500$  (for **MOD2**  $c_{11}=c_5, c_{12}=c_6, c_{13}=c_9, c_{14}=c_{10}$ ); Variant D: the external force is perpendicular to the gravity force of the hand, the weight coefficients are:  $c_1=1500, c_2=-0.0015, c_3=1500, c_4=-0.0015, c_5=0.5, c_6=-1500, c_7=0.15, c_8=-1500, c_9=0.15, c_{10}=-1500$  (for **MOD2**  $c_{11}=c_5, c_{12}=c_6, c_{13}=c_9, c_{14}=c_{10}$ ).

**Variant B.**  $M_{sh} > 0$ ,  $M_{cl} > 0$ ,  $M_{wr} < 0$

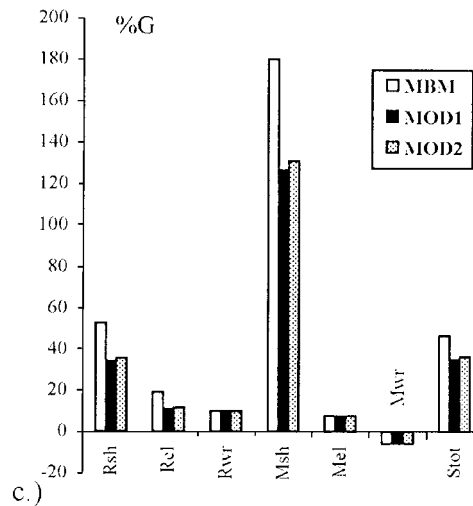
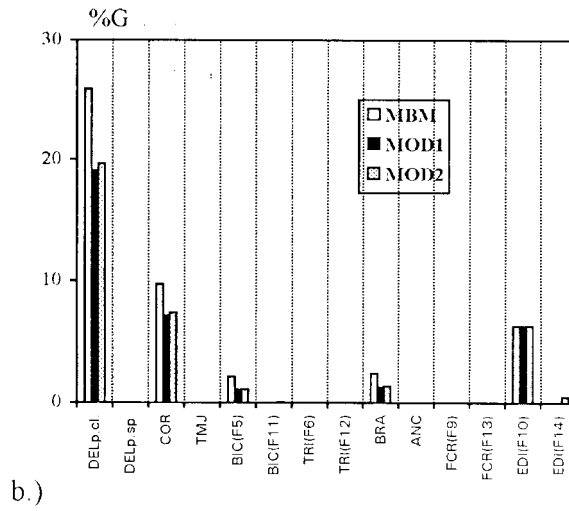
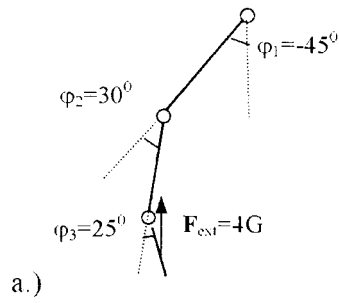


FIGURE 6 (Continued).

**Variant C.  $M_{sh}>0, M_{cl}<0, M_{wr}>0$**

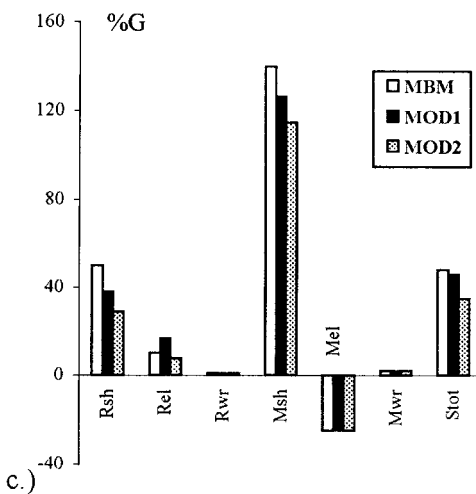
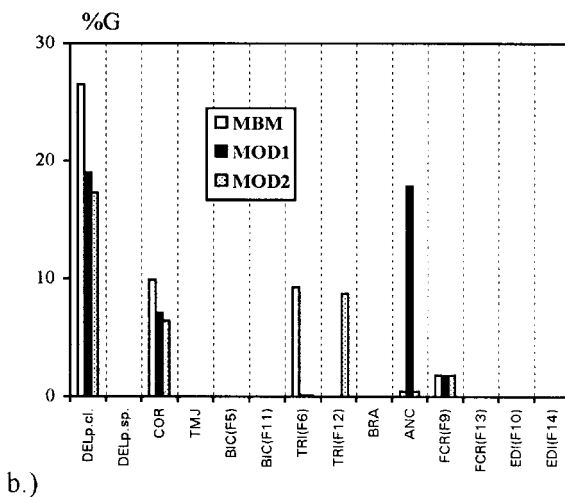
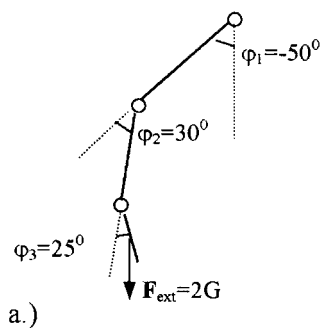


FIGURE 6 (Continued).



**Variant D.**  $M_{sh} < 0, M_{el} > 0, M_{wr} > 0$

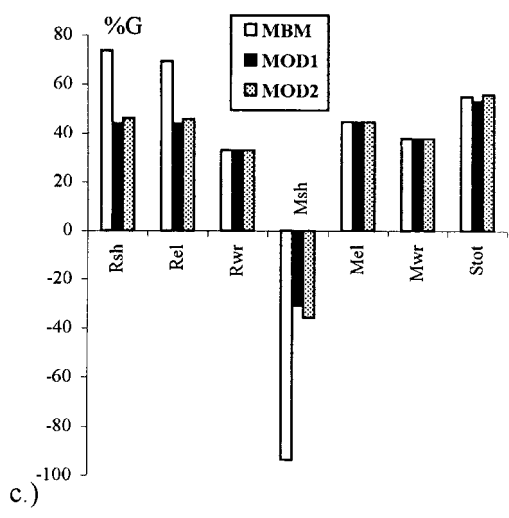
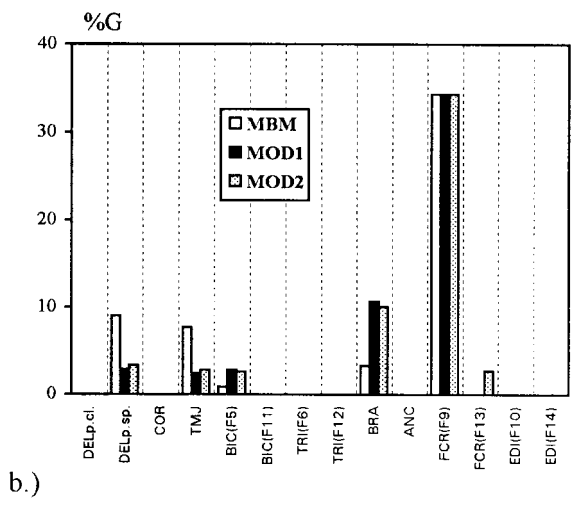
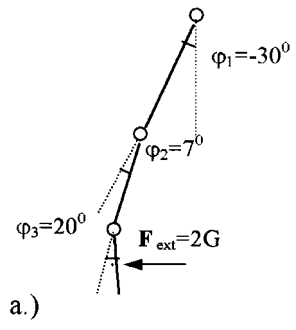


FIGURE 6 (Continued).

and D). From top to bottom there are shown: (a) the configuration of the models (the chosen joint angles,  $\varphi_i$ , and external force applied to the hand,  $F_{\text{ext}}$ ); (b) the predicted muscle forces compared between the models; (c) the computed joint reactions and moments.  $M_{\text{sh}}$ ,  $M_{\text{el}}$  and  $M_{\text{wr}}$  are the total muscle moments in the shoulder, elbow and wrist joint respectively. They have magnitudes equal to the net external moments in the respective joints, but opposite a direction (see Appendix). The directions of the moments fall into these four variants because of the chosen external forces and weight coefficients of the muscles (*i.e.*, their forces) that in turn reflects the joint reactions and moments. For the configurations shown in Figure 6, the biarticular muscles in the basic model can be chosen as active or inactive, depending on the weight factors of their forces in the objective function. For example, since the moments in the shoulder and elbow have different direction for Variant A, the forces of the muscles BIC and TRI might be nearly zero (then only the monoarticular muscles will work in the shoulder and elbow) or essentially different from zero (in this case these biarticular muscles will help monoarticular muscles in one of the joint, but in the other joint their forces will increase the net external moment). Identical weight factors were chosen for the three models such that possible solutions of the optimization task exist and the one-joint muscles whose moments add to the respective net external moment be inactive.

For the first case (**Variant A**) m.BIC is chosen to be non-active ( $c_5 = 250$ ), whereas the extensor m.TRI is chosen to be active ( $c_6 = c_{12} = -0.005$ ).  $M_{\text{wr}}$  is with clockwise direction (*i.e.*,  $M_{\text{wr}} < 0$ ) because of action of  $F_{\text{ext}}$ , so the wrist extensors will be primary active. Since the reaction in the wrist has bigger counterclockwise moment with respect to  $O_2$  than the clockwise moment of the gravity force of the forearm, the net external moment in the elbow is with counterclockwise direction. Hence  $M_{\text{el}} < 0$ . The reaction in the elbow, however, has such direction (from  $O_2$  to  $O_3$  upwards and from the left side of  $O_2O_3$ ) that its moment with respect to  $O_1$  coincides with the

moment of  $G_1$  (see Fig. 1a) and this leads to a clockwise net external moment in the shoulder (*i.e.*,  $M_{\text{sh}} > 0$ ). The equilibrium in the shoulder joint is maintained by the actions of the one-joint flexors DELp.cl. and COR only (without a help of the biarticular m.BIC). Their predicted forces are the biggest for the model with biarticular muscles (**MBM**) - see Figure 6b (Variant A). A great force was predicted in m.ANC for **MOD1**. The reason is that this muscle is the only one performing extension in the elbow. When m.TRI is modelled as a biarticular one however (**MBM**), the net external moment in the elbow joint is countered by this muscle, but not by the monoarticular m.ANC, regardless of the fact that its predicted force will increase  $M_{\text{sh}}$ . When m.TRI is modelled by two one-joint muscles (**MOD2**), the predicted force  $F_{12}$ , which represents the action of m.TRI in the elbow joint is much bigger than the force of m.ANC because of the proportion between the weight factors of these muscles ( $c_6 = c_{12} = -0.005$  and  $c_8 = -0.01$ ) [41].

For **Variant B**  $M_{\text{wr}} < 0$  because of the applied external force, but  $M_{\text{el}} > 0$  because of the direction of the reaction in the wrist joint (it is from the left side of  $O_2O_3$  with direction from  $O_3$  to  $O_2$  upwards - see Fig. 1a). The direction of the moment of  $R_{\text{wr}}$  with respect to  $O_2$  is clockwise one. It is opposite to this of  $G_2$ , but its value is much bigger. Hence  $M_{\text{el}} > 0$ . The same refers to the reaction in the elbow joint - its moment with respect to  $O_1$  has opposite direction to the direction of the moment of  $G_1$ , but its value is bigger (hence  $M_{\text{sh}} > 0$ ). The behavior of the muscles FCR and EDI are the same for the three models since they are the only muscles which drive the wrist joint. The equal predicted forces of m.EDI, however, are transferred *via* the next joints in different ways for the three models. The forces of the muscles BIC, BRA, COR and DELp.cl. are the biggest for the model with biarticular muscles (Fig. 6b, Variant B). This is due to the increase in  $R_{\text{el}}$  because of the action of m.EDI in the elbow joint concerning **MBM** and **MOD2** (see Fig. 6c, Variant B). As a sequence

more muscle forces are needed for equilibrium of the shoulder joint.

For the next situation (**Variant C**), the net external moment  $M_{el}$  in the middle (elbow) joint has a direction opposite to the directions of  $M_{sh}$  and  $M_{wr}$ . This happens because  $R_{wr}$  passes from the right hand of  $O_3K$ , downwards, and the direction of its moments with respect to  $O_2$  coincides with this of  $G_2$  (see Fig. 1a). The direction of these moments is counterclockwise one, hence  $M_{el} < 0$ .  $R_{el}$  is nearly along  $O_2O_3$ , but is with direction from  $O_2$  upwards. Its moment with respect to  $O_1$  is with clockwise direction and is opposite to this of  $G_1$ . The value of the reaction moment, however, is much bigger than this of the gravity force, so  $M_{sh} > 0$ . Here all four biarticular muscles have opposite actions in the joints they serve. The force of m.FCR, however, must be greater than that of m.EDI, since no monoarticular muscles driving the wrist joint are included in the model and  $M_{wr} > 0$ . The equal predicted forces of m.FCR for the three models affect the situation in the next joints in different way. The forces predicted in m.DELp.cl. and COR,  $R_{sh}$  and  $M_{sh}$  considerable increase for the model with biarticular muscles (Figs. 6b and c, Variant C). A big force is predicted in the monoarticular muscle ANC for **MOD1**, while for the model with biarticular muscles, m.TRI is preferred, independently from the fact that its force will give rise in the net external moment in the shoulder (see Figs. 6b and c, Variant C). Hence, even for such situation when the moment direction of a biarticular muscle coincides with the direction of the net external moment of one joint, but is opposite to the direction of the net external moment in the other joint, the biarticular muscle is preferred against the monoarticular one (*i.e.*, TRI *versus* ANC in **MBM**).

The last example (**Variant D**) shows an interesting model configuration where  $M_{sh} < 0$  for the first time and the muscles driving the wrist joint are more active than those driving the shoulder joint.  $M_{wr} > 0$  because of action of a horizontal force applied to the hand (Fig. 6a, Variant D). The

reaction in the wrist joint is upward from  $O_3$  and passes from the left hand of  $O_3O_2$ , so its moment direction with respect to  $O_2$  is clockwise one, while the direction of the moment of  $G_2$  is clockwise one (see Fig. 1a). The moment of  $R_{wr}$  is much bigger than this of  $G_2$ , however. This is why the net external moment in the elbow has clockwise direction.  $R_{el}$ , however, is from the right hand of  $O_2O_1$ , upwards. Its moment direction with respect to  $O_1$  coincides with this of  $G_1$  and is counterclockwise, hence  $M_{sh} < 0$ . It is supposed here that m.TRI is inactive ( $c_6 = -1500$ ) and m.BIC is active ( $c_5 = 0.5$ ) but less than m.BRA ( $c_7 = 0.15$ ). Independently of the less predicted force in the biarticular BIC for **MBM** (Fig. 6b, Variant D), it influences much the loading of the shoulder joint.  $M_{sh}$ ,  $R_{sh}$ ,  $R_{el}$  and the predicted forces of one-joint shoulder extensors TMJ and DELp.sp. are the biggest for **MBM** (Figs. 6b and c, Variant D).

As it can be seen from the lower two plots for the four variants in Figure 6, no uniform conclusion about the advantages of the two-joint muscles could be made. For most of the shown by figures and other similar investigated cases, the sum of all muscle forces,  $S_{tot}$ , was the biggest for the model with biarticular muscles **MBM** compared to **MOD1** and **MOD2**. The same refers to the joint reactions and moments. Opposite cases could also take place, however.

Aiming to investigate how the transition of a two-joint muscle from an active state (with considerable predicted force, respectively with closer to zero weight coefficient) to an inactive one (with predicted force nearly zero, respectively with big module of the weight coefficient) influences the behaviors of the models, the output parameters were traced out by changing the weight coefficients of these muscles. For **Variant A**,  $M_{sh}$  and  $M_{el}$  had opposite direction, m.BIC was chosen as inactive ( $c_5 = 250$ ) and m.TRI was chosen as active ( $c_6 = -0.005$ ). Increasing the force of m.BIC by reducing its weight coefficient (remember that  $c_{11} = c_5$  for **MOD2**), all output parameters increase (see Fig. 7) except the force of m.EDI,  $M_{el}$ ,  $M_{wr}$  and  $R_{wr}$  which remain nearly unchanged.

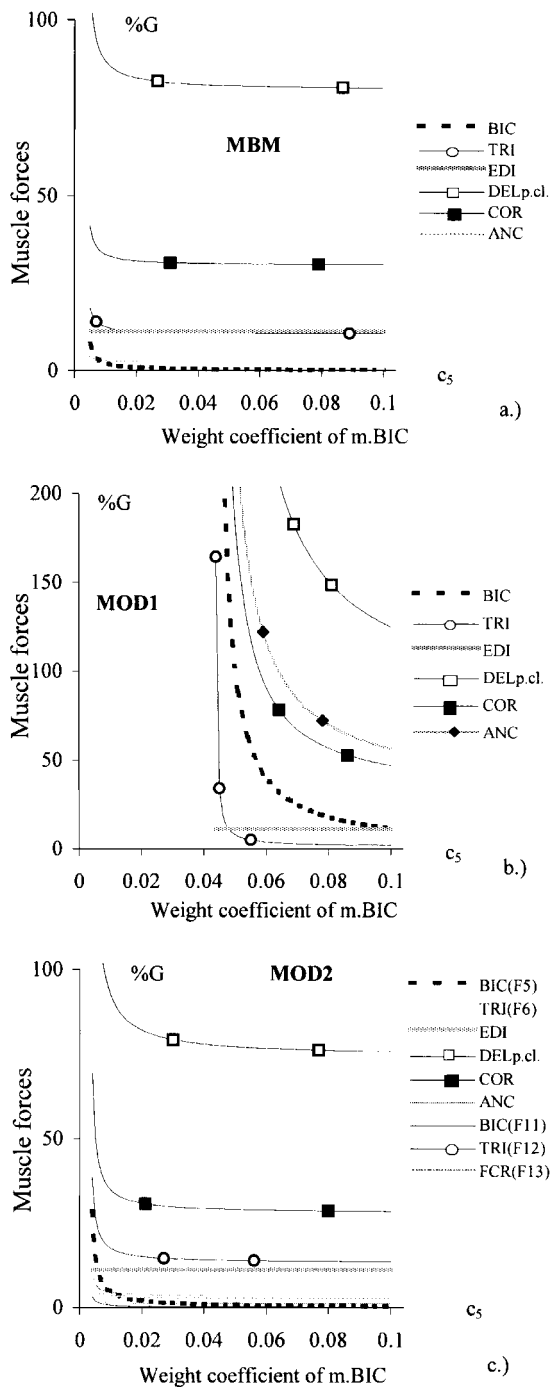


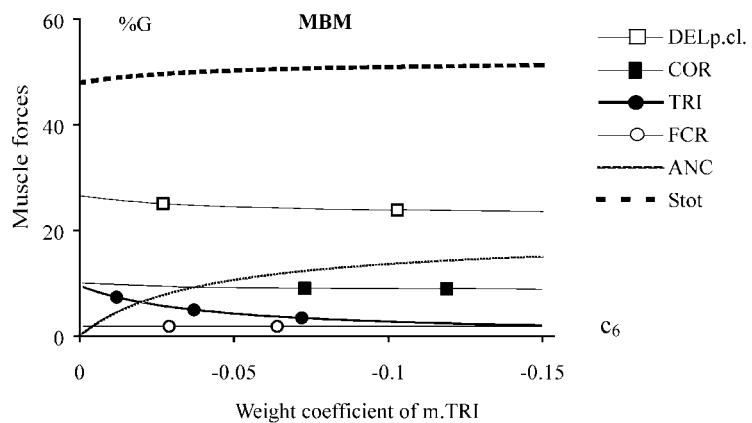
FIGURE 7 Comparison of the predicted muscle forces in-between the three models when the weight factor of the muscle BIC,  $c_5$ , is changed (note that  $c_{11} = c_5$  for MOD2). The used parameters are the same as these for Figure 6 (Variant A), only essentially different from zero muscle forces are shown (note that the scale of Fig. 7b is different from the scale of Figs. 7a and 7c). The muscle forces are computed as relative to forearm weigh G.

The change of the output parameters of the **MOD1**, however, is much faster and stronger (Fig. 7b). No possible solutions exist for  $c_5 < 0.044$ . For **MBM** (Fig. 7a) and **MOD2** (Fig. 7c), the influence of  $c_5$  becomes significant for values less than 0.01. The predicted parameters for **MBM** undergo the least changes, the control of two-joint muscle **BIC** is graceful, but its force can not increase too much. The behavior of the **MOD2** is similar to **MBM** because  $c_5 = c_{11}$ . Increasing the force of the biarticular m.**BIC** in **MBM** (respectively  $F_5$  and  $F_{11}$  for **MOD2**) it was expected that the forces of monoarticular shoulder flexors will decrease because of additional help of m.**BIC**. This does not happen, however. Obviously the antagonistic action of m.**BIC** in the elbow joint is stronger because of the increase in the reactions in this joint. The change of the weight coefficient of m.**TRI** hardly influences the output parameters. Only a redistribution between the forces of the muscles **TRI** and **ANC** for **MBM**, and  $F_{12}$  and **ANC** for **MOD2** was observed (not shown in the figures, but the plots of this redistribution are similar to those shown in Figs. 8a and 8c).

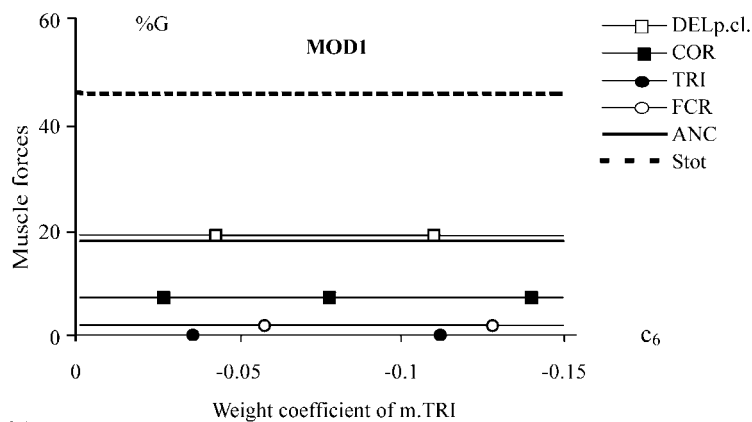
The net external moments in the elbow and wrist joints had opposite directions for the configuration shown in Figure 6, **Variant B**, hence the behavior of m.**EDI** and m.**FCR** are of interest. Muscle **EDI** was chosen as active ( $c_{10} = -0.5$ ) while m.**FCR** was inactive ( $c_9 = 1500$ ). Changing  $c_9$  from 10 to 0, *i.e.*, increasing the predicted force of m.**FCR**, the behavior of the three models was similar - all output parameters increased (not shown in the figure). The increase of the antagonistic activity of m.**FCR** in the wrist, however, can not be too large. No positive solutions exist for  $c_9 < 0.7$  for the three models. The identical influence of  $c_9$  on the output parameters of the three models in this case is due to the assumption that only two biarticular muscles act in the wrist, *i.e.*, there are no monoarticular muscles that may take up the effort. The output parameters of **MBM** and **MOD2** do not change when the weight coefficient of m.**EDI**,  $c_{10}$ , changes from  $-1$  to zero, because this muscle force is predicted mainly to ensure the equilibrium

in the wrist joint. The behavior of **MOD2** was different, however, due to the increase in the force  $F_{14}$  (remember that  $c_{14} = c_{10}$ ) and the addition of its moment to the net external moment in the elbow joint. The case shown in Figure 6, **Variant C**, is the most interesting one. Here all two-joint muscles might be either active or inactive. Since there is only one muscle performing flexion in the wrist, **FCR**, its predicted force is equal in the three models. All output parameters remain nearly constant changing the weight coefficient of this muscle (not shown in the figure). Increasing the force of m.**EDI** by approaching its weight coefficient to zero, all output parameters increase, because of the antagonistic action of this muscle in the wrist joint (not shown in the figure). Hence, the addition of the moment of this muscle in the elbow joint does not help much m.**TRI** and m.**ANC** in maintaining the equilibrium in this joint. The plots of the influence of the weight coefficient of m.**BIC** for this variant are similar to those shown in Figure 7. Figures 8 and 9 show the results obtained when the weight coefficient of m.**TRI**,  $c_6$ , is changed. The output parameters for **MOD1** remain nearly constants when changing the control on the activity of m.**TRI**. As to **MBM** and **MOD2**, when  $c_6$  goes close to zero, the differences occur in the behavior of the muscles **DELp.cl.** and **COR** (Figs. 8a and c). Their forces increase for **MBM**, but decrease for **MOD2**. Here a reciprocal activation of m.**TRI** and m.**ANC** (respectively  $F_{12}$  and **ANC** for **MOD2**) is observed. The dependencies of the joint moment and reactions on  $c_6$  are also different for **MBM** and **MOD2** (see Fig. 9). When m.**TRI** becomes more active,  $R_{sh}$  increases for **MBM**, but it decreases for **MOD2**. From point of view of the total muscle force (see  $S_{tot}$  in Fig. 8a), it is more effective if the biarticular m.**TRI** is with more predicted force, independently of the fact that he works as antagonist in one of the joints (shoulder in the current case). An opposite situation was observed in Figure 4b.

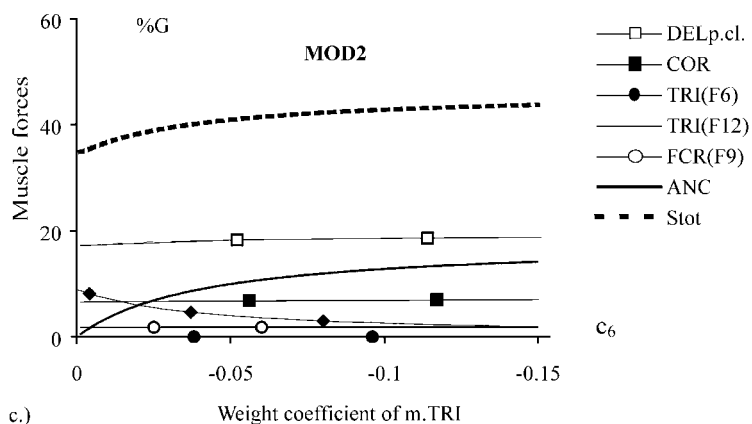
For **Variant D** (Fig. 6), a redistribution of the efforts between the muscles **BIC** and **BRA** was



a.)



b.)



c.)

FIGURE 8 Comparison of the predicted muscle forces in-between the three models when the weight factor of the muscle TRI,  $c_6$ , is changed (note that  $c_{12} = c_6$  for MOD2). The used parameters are the same as these for Figure 6 (Variant C), only essentially different from zero muscle forces are shown,  $S_{tot}$  - sum of all muscle forces. The forces are computed as relative to forearm weigh G.

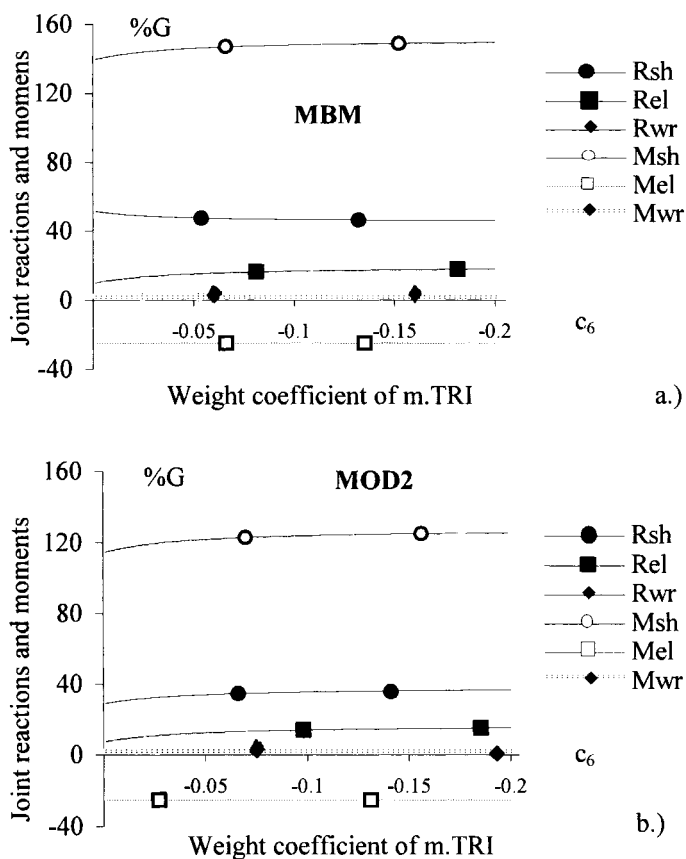


FIGURE 9 Comparison of the predicted muscle moments in the shoulder,  $M_{sh}$ , elbow,  $M_{el}$ , and wrist,  $M_{wr}$ , joints and joint reactions ( $R_{sh}$ ,  $R_{el}$ ,  $R_{wr}$ ) in-between **MBM** and **MOD2** when the weight factor of the muscle TRI,  $c_6$ , is changed (note that  $c_{12} = c_6$  for **MOD2**). The used parameters are the same as these for Figure 6 (Variant C). The joint reactions are computed as percentage of forearm weight  $G$ , the length unit for moment calculations is centimeter.

observed changing the weight coefficient of m.BIC from 1 to zero (not shown in the figure). The increase of the force of m.BIC causes  $S_{tot}$  and  $M_{sh}$  to increase. The change of the output parameters, however, is very graceful for **MBM**.  $c_5$  may be practically zero for the model with biarticular muscles, whereas no positive solutions exist for **MOD1** if  $c_5 < 0.062$  and for **MOD2** if  $c_5 < 0.052$ . The weight coefficient of the other muscle of interest m.TRI (remember that  $M_{sh} < 0$  and  $M_{el} > 0$ ) does not influence the output parameters of **MOD1**, whereas the behavior of **MBM** and **MOD2** depends on its value. Changing the state of m.TRI from inactive to active (by increasing  $c_6$  from  $-1500$  to zero)  $S_{tot}$ ,  $R_{sh}$ ,  $R_{el}$  and  $M_{sh}$

increase rapidly. Hence, the antagonistic action of m.TRI in the elbow joint influences greatly the situation in the shoulder joint when this muscle is presented as biarticular one.

#### 4. CONCLUSIONS

On the basis of the performed numerical experiments whose aim was to compare the model with biarticular muscles and its two modifications with only monoarticular TRIs, the following conclusions are drawn:

- The findings of other authors [23] was confirmed that it is not possible to formulate strictly

advantages of the biarticular muscles. Their features depend on limb position, external loading, activity of other muscles, neural control, *etc.*

- Concerning the muscles of the upper limb, it could be stated that the monoarticular muscles are more powerful than the biarticular ones (see also [42]). Almost always the predicted forces of the muscles BIC and TRI were less than those of their respective monoarticular synergists. This happens because in most situations, the moments in the proximal joints are greater than in the distal ones. The upper limb is an open kinematic chain and every link bears the weight of the previous links. For example, since the net external moment in the shoulder joint is usually bigger than in the elbow joint in case when these moments have clockwise direction, the force of m.BIC is predicted primarily to ensure the equilibrium in the elbow joint, but not in the shoulder joint. Otherwise, antagonistic co-contraction of the elbow extensors will be necessary, which is not effective. Such anatomical structure (the monoarticular muscles are more powerful and bigger and are situated around more proximal joints) leads to a shift of the actuator mass to the trunk. In such way the mass, geometrical and inertial characteristics of the end link (hand) improves and thus more precise positioning is achieved.
- The conclusion of other authors [10, 27] was confirmed that bi-articular muscles fine-tune muscle coordination. Supposing that the vector of the weight coefficient could be treated as a control vector, it was concluded that for a current limb position much more sets of  $c_i$ -s giving possible solutions of the indeterminate problem exist for the model with biarticular muscles in comparison with the two other models with only monoarticular muscles. Hence, the possibilities for the achievement of equilibrium in the joints through different levels of the muscle forces, respectively with different control strategies, are much wider for the model with biarticular muscles. On the other hand,

fixing all input parameters of the models and changing only a weight coefficient of a muscle, *i.e.*, changing smoothly the predicted force of this muscle, it was observed that the possible  $c_i$ 's range is much larger for a two-joint muscle than for a monoarticular one. The change of  $c_i$  of a monoarticular muscle causes much faster changes in its predicted force, whereas when changing  $c_i$  of a biarticular muscle, the force changes are much finer and smoother.

- Contrary to the expectation that the presence of biarticular muscles will be more effective (from point of view of the total necessary force for a particular motor task),  $S_{tot}$  was the biggest for the model with biarticular muscles for most model's configurations. For other configurations, however, this sum was the smallest (see Figs. 3d, 4b and the lowest figures of all variants in Fig. 6). It was reasonable to expect that less total muscle force will be needed when biarticular muscles are presented in the model, because one force (the force of a biarticular muscle) would develop moments with respect to two joints. The reason for occasionally opposite results is the influence of the joint reactions. The action of the biarticular muscle in the distal joint usually increases the reaction in this joint, thus the net external joint moment in the proximal joint increases too.
- The presence of two-joint muscles does not induce by itself antagonistic co-contraction (see Figs. 3e and 5a), neither in the cases when all joint moments have identical directions nor when some joint moments have opposite directions to others (see also [42]).
- The pattern of muscle force's distributions does not differ significantly between the models, hence it does not depend on the way the biarticular muscles are modelled, but rather on the weight factors  $c_i$ , *i.e.*, on motor control. Greater differences between the models were observed concerning the reactions and moments in the shoulder and elbow joints. In general, these parameters were the biggest for the model with biarticular muscles. Since as these quantities



reflect the forces with which the previous body acts on the next body, this leads to the following conclusion: the presence of biarticular muscles increases the stability of the joints. They become stiffer. Thus the limb is protected from fractures and a stable base for manipulation is provided.

- Comparing the result obtained for **MBM** and **MOD2** the following must be pointed out. The weight coefficients of the two forces with which the action of a biarticular muscle in the two joints was presented in **MOD2** were set always equal during the numerical experiments, *i.e.*,  $c_{11} = c_5$ ,  $c_{12} = c_6$ ,  $c_{13} = c_9$ , and  $c_{14} = c_{10}$ . This was done aiming to provide equal control for the three models, hence the obtained results should be comparable between the models. The results from the numerical experiments showed that  $F_{11}$  is never equal to  $F_5$ . The same refers to  $F_{12}$  and  $F_6$ ,  $F_{13}$  and  $F_9$ ,  $F_{14}$  and  $F_{10}$ . The main corollary is that the predicted muscle force depends essentially on the magnitude of the net external joint moments.
- The role of the two-joint muscles is different for cases when all net external moments have identical directions in comparison with cases when some of these moments has opposite direction to others. For the first case, the presence of the biarticular muscles leads to an increase in the joint moments and reactions. For the second case, their control allow a graceful transition from active to non-active state. This leads to a redistribution of the efforts between the two neighbor joints.
- Fixing all input parameters identical for the three models and increasing the external force applied to the hand, all output parameters increase proportionally to the external force, *i.e.*, the motor control strategy does not change by itself. No redistribution of the efforts between the muscles was observed. This fact suggests that the weight coefficients in the objective functions must not be constant. Hence, if the two-joint muscles have an advantage in carrying extra weight, the reason could be searched in their control.

## 5. FINAL NOTES

The present study does not claim to be an exhaustive one. The possibilities for performing different numerical experiments are tremendous. The basic model used for the present study is of course simplified. Some of the muscles performing flexio and extensio in the shoulder, elbow and wrist joints are omitted. This was found to be a defect with respect to the investigation of the effort distribution in the wrist. The investigation is performed for quasistatistical conditions only. The peculiarities of the biarticular muscles in dynamics can probably be different. Differences exist also between the behaviour of the biarticular muscles of the upper and the lower limb. Hence, the conclusions made in the present paper could not be transferred mechanically for other models and other, not investigated by numerical experiments, motor acts.

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## APPENDIX

Equations for equilibrium for the three models (these equations are written in the local coordinate systems - see Figure 1a for definition of the coordinate systems and Figure 1c for definition of the parameters of the muscle forces  $\mathbf{ds}_i$ ,  $\mathbf{de}_i$ ,  $\mathbf{dw}_i$  and  $\alpha_{i,j}$ ):

### HAND (body 3)

$$\sum_i \mathbf{dw}_i F_i = M_{O_3}(G_3) + M_{O_3}(F_{\text{ext}}), \quad (1)$$

$$\sum_i -\sin(\alpha_{i,3}) F_i = G_3^{(X_3)} + F_{\text{ext}}^{(X_3)} + R_{23}^{(X_3)}, \quad (2)$$

$$\sum_i \cos(\alpha_{i,3}) F_i = G_3^{(Y_3)} + F_{\text{ext}}^{(Y_3)} + R_{23}^{(Y_3)}. \quad (3)$$

The summation is made for  $i=9\div 10$  for the three models.

### FOREARM (body 2)

$$\sum_i \mathbf{de}_i F_i = M_{O_2}(G_2) + M_{O_2}(R_{32}), \quad (4)$$

$$\sum_i -\sin(\alpha_{i,2}) F_i = G_2^{(X_2)} + R_{32}^{(X_2)} + R_{12}^{(X_2)}, \quad (5)$$

$$\sum_i \cos(\alpha_{i,2}) F_i = G_2^{(Y_2)} + R_{32}^{(Y_2)} + R_{12}^{(Y_2)}. \quad (6)$$

The summation is made for:  $i=5\div 10$  for **MBM**;  $i=5,7,8$  for **MOD1**;  $i=5, 7, 8, 12, 13, 14$  for **MOD2** and  $\mathbf{de}_{12} = \mathbf{de}_6$ ,  $\mathbf{de}_{13} = \mathbf{de}_9$ ,  $\mathbf{de}_{14} = \mathbf{de}_{10}$ ,  $\alpha_{12,2} = \alpha_{6,2}$ ,  $\alpha_{13,2} = \alpha_{9,2}$ ,  $\alpha_{14,2} = \alpha_{10,2}$ .

### ARM (body 1)

$$\sum_i \mathbf{ds}_i F_i = M_{O_1}(G_1) + M_{O_1}(R_{21}), \quad (7)$$

$$\sum_i \cos(\alpha_{i,1}) F_i = G_1^{(X_1)} + R_{21}^{(X_1)} + R_{01}^{(X_1)}, \quad (8)$$

$$\sum_i \sin(\alpha_{i,1}) F_i = G_1^{(Y_1)} + R_{21}^{(Y_1)} + R_{01}^{(Y_1)}. \quad (9)$$

The summation is made for:  $i=1\div 6$  for **MBM**;  $i=1, 2, 3, 4, 6$  for **MOD1**;  $i=1, 2, 3, 4, 6, 11$  for **MOD2** and  $\mathbf{ds}_{11} = \mathbf{ds}_5$ ,  $\alpha_{11,1} = \alpha_{5,1}$ .

$M_{O_j}(G_j)$  ( $j=1, 2, 3$ ) is the moment of the gravity force of the  $j$ -th body ( $j=1$  arm,  $j=2$  - forearm,  $j=3$  - hand; see Fig. 1a) with respect to the rotation center  $O_j$ .  $G_j^{(X_j)}$  and  $G_j^{(Y_j)}$  are the component of the gravity force  $G_j$  along the coordinate axes  $O_j X_j$  and  $O_j Y_j$  respectively.

$M_{O_3}(F_{\text{ext}})$ ,  $F_{\text{ext}}^{(X_3)}$ ,  $F_{\text{ext}}^{(Y_3)}$  are the moment of an external force applied to the hand with respect to the center  $O_3$  and the components of this force along the coordinate axes  $O_3 X_3$  and  $O_3 Y_3$  respectively.

$R_{ij}$  is the force with which the  $i$ -the body acts on the  $j$ -th body (joint reaction). Note that  $R_{32}$  and  $R_{23}$  (as well as their components along the axes of the coordinate systems) have equal magnitudes but opposite directions. The same refers to  $R_{12}$  and

$R_{21}$ .  $\sqrt{(R_{01}^{(X_1)})^2 + (R_{01}^{(Y_1)})^2}$  is the reaction in the shoulder joint and is denoted by  $R_{sh}$ ,

$\sqrt{(R_{12}^{(X_2)})^2 + (R_{12}^{(Y_2)})^2}$  is the reaction in the elbow

joint and is denoted by  $R_{el}$ ,  $\sqrt{(R_{23}^{(X_3)})^2 + (R_{23}^{(Y_3)})^2}$  is the reaction in the wrist joint and is denoted by  $R_{wr}$ , where  $R_{23}^{(X_3)}$ ,  $R_{23}^{(Y_3)}$ ,  $R_{12}^{(X_2)}$ ,  $R_{12}^{(Y_2)}$ ,  $R_{01}^{(X_1)}$ ,

$R_{01}^{(Y_1)}$  are the respective components along the axes of the coordinate systems.  $M_{O_2}(R_{32})$  and  $M_{O_1}(R_{21})$  are the moments of the respective joint reactions with respect to the centers  $O_2$  and  $O_1$ .

The left side members of the moment Eqs. (1), (4) and (7), *i.e.*,  $\sum_i d\mathbf{w}_i F_i$ ,  $\sum_i d\mathbf{l}_i F_i$  and  $\sum_i d\mathbf{s}_i F_i$ , are called total muscle moments in the wrist, elbow and shoulder joints respectively and are denoted by  $M_{wr}$ ,  $M_{el}$  and  $M_{sh}$ . The expressions from the right

side of these equations are called net external moments in the respective joints. The net external joint moment is the moment of the gravity (and external force for body 3), but it includes also the moment of reaction in the previous joint (for bodies 1 and 2). It has a magnitude equal to the total muscle moment in the respective joint, but an opposite direction.