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Sensitivity of predicted muscle forces to parameters of the optimization-based human leg model revealed by analytical and numerical analyses

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Abstract

There are different opinions in the literature on whether the cost functions: the sum of muscle stresses squared and the sum of muscle stresses cubed, can reasonably predict muscle forces in humans. One potential reason for the discrepancy in the results could be that different authors use different sets of model parameters which could substantially affect forces predicted by optimization-based models. In this study, the sensitivity of the optimal solution obtained by minimizing the above cost functions for a planar three degrees-of-freedom (DOF) model of the leg with nine muscles was investigated analytically for the quadratic function and numerically for the cubic function. Analytical results revealed that, generally, the non-zero optimal force of each muscle depends in a very complex non-linear way on moments at all three joints and moment arms and physiological cross-sectional areas (PCSAs) of all muscles. Deviations of the model parameters (moment arms and PCSAs) from their nominal values within a physiologically feasible range affected not only the magnitude of the forces predicted by both criteria, but also the number of non-zero forces in the optimal solution and the combination of muscles with non-zero predicted forces. Muscle force magnitudes calculated by both criteria were similar. They could change several times as model parameters changed, whereas patterns of muscle forces were typically not as sensitive. It is concluded that different opinions in the literature about the behavior of optimization-based models can be potentially explained by differences in employed model parameters. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Indeterminate problem; Sensitivity analysis; Static optimization; Lower limb; Muscle force

1. Introduction

Individual muscle forces produced by humans in every-day tasks and occupational and athletic activities are important quantities; the knowledge of which has numerous applications in human biology, orthopedics, and motor control. Methods of mathematical modeling, including static optimization, are used to estimate individual muscle forces in the human body because their direct measurements are difficult to make. The accuracy of muscle force predictions depends upon the objective function employed in static optimization and model parameters as well as other factors (model assumptions, number of degrees-of-freedom (DOF) and modeled muscles, etc.).

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Two particular objective functions, the sum of muscle stresses squared and the sum of muscle stresses cubed, have often been employed in recent years and, according to some authors, predict muscle forces that correlate reasonably well with muscle activation in selected tasks (Crowninshield and Brand, 1981; Hughes et al., 1994; Prilutsky et al., 1998; van Bolhuis and Gielen, 1999; van Dieen, 1997). Other authors, however, report that these or similar criteria do not predict magnitudes and patterns of muscle forces satisfactorily (Buchanan and Shreeve, 1996; Dul et al., 1984; Glitsch and Baumann, 1997; Karlsson and Peterson, 1992; van Der Helm, 1994). Possible reasons for these contradictions are that (i) calculated muscle forces are very sensitive to model parameters (moment arms and physiological cross-sectional areas (PCSAs)) (Brand et al., 1986; Dul et al., 1984; Nussbaum et al., 1995; Raikova, 1996, 2000b) and (ii) different authors use different sets of model parameters.

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To resolve this issue, the sensitivity of the optimal solution to model parameters should be investigated for the model of interest and, preferably, analytical relationships between predicted muscle forces and model parameters should be obtained so that the effect of model parameters can be clearly seen. Analytical relationships between optimal forces and model parameters have mainly been obtained for simplified 1DOF models (Challis and Kerwin, 1993; Dul et al., 1984; Raikova, 1996; Zatsiorsky et al., 1998). Analytical solutions for 2DOF or 3DOF models were reported by Herzog and Binding (1992, 1993) and Raikova (2000a). The models of Herzog and Binding, however, were considerably simplified by assuming equal moment arms and PCSAs of all muscles. Raikova (2000a) did not examine the two objective functions that are of interest in the present study. The sensitivity analyses of optimal muscle forces, for more realistic multi-DOF models of the spine (Nussbaum et al., 1995) and the arm (Hughes and An, 1997; Raikova, 2000b), were conducted, primarily, numerically and demonstrated a pronounced effect of model parameters. The influence of variations in PCSAs on predicted forces of the lower limb has also been studied (Brand et al., 1986), but not systematically.

Therefore, the purposes of this study were (i) to attempt to find analytical relationships between model parameters and muscle forces minimizing the sum of muscle stresses squared and the sum of muscle stresses cubed for a rather realistic 3DOF planar model of the human leg, and (ii) to investigate systematically the sensitivity of the optimal solution to model parameters using the analytical approach and numerical optimization.

2. Methods

2.1. Optimization problem formulation and solution

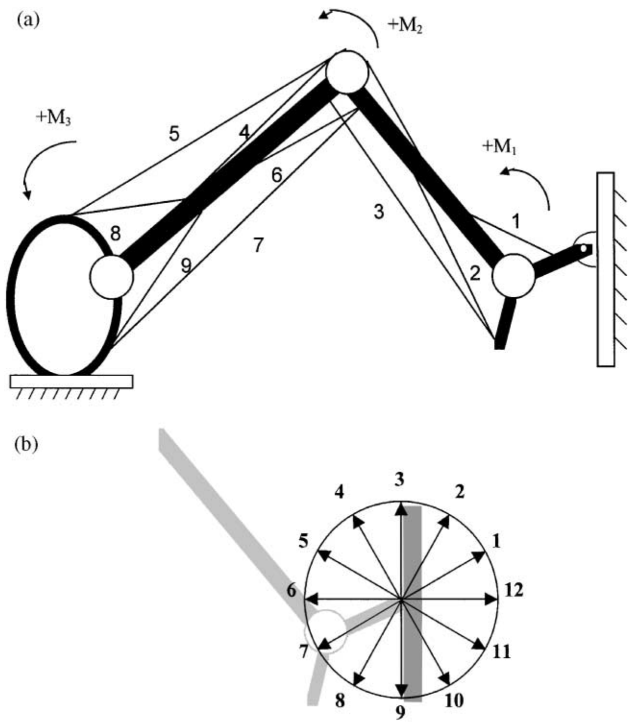
Consider a planar 3DOF model of the human lower limb with nine major muscles (Fig. 1a). The limb posture resembles the subjects' posture in the experiments of Wells and Evans (1987), a description of which is given later in the text. The individual muscle forces that produce given joint moments can be found by solving the following static optimization problem:

$$\text{minimize } Z = \sum_{i=1}^{9} \left(\frac{F_i}{A_i}\right)^n \quad n = 2, 3, \tag{1}$$

subject to:

$$f_1 = d_1F_1 - d_2F_2 - d_{3\mathrm{a}}F_3 - M_1 = 0, \tag{2.1}$$

$$f_2 = -d_{3\mathrm{k}}F_3 + d_4F_4 + d_{5\mathrm{k}}F_5 - d_6F_6 - d_{7\mathrm{k}}F_7 - M_2 = 0, \tag{2.2}$$



	Dir1	Dir2	Dir3	Dir4	Dir5	Dir6	Dir7	Dir8	Dir9	Dir10	Dir11	Dir12
M_1	4	14	20	17	14	6	-3	-14	-19	-21	-15	-5
M_2	33	46	46	33	19	-9	-28	-44	-45	-38	-13	13
M_3	31	59	70	59	44	7	-24	-52	-70	-72	-41	-3

Fig. 1. A schematic representation (view from above) of the employed two-dimensional model of the lower limb. (a) The limb posture resembles that of the experiments of Wells and Evans (1987) (their Fig. 4). The model has three joints (hip, knee and ankle) which are crossed by muscles exerting forces F_i ($i = 1, 2, \ldots, 9$): 1, tibialis anterior (TA, ankle flexor); 2, soleus (SO, ankle extensor); 3, gastrocnemius (GA, ankle extensor, knee flexor); 4, vastii (VA, knee extensor); 5, rectus femoris (RF, knee extensor, hip flexor); 6, short head of biceps femoris (BFS, knee flexor); 7, long head of biceps femoris (BFL, knee flexor, hip extensor); 8, iliacus (IL, hip flexor); and 9, gluteus maximum (GLM, hip extensor). Joint moments (M_1, M_2 and M_3; the chosen positive directions are shown in the figure), PCSAs, and muscle moment arms with respect to the joint centers were assumed known. The nominal values of the muscle moment arms (in m) were: $d_1 = 0.0298$, $d_2 = 0.0440$, $d_{3\mathrm{a}} = 0.044$, $d_{3\mathrm{k}} = 0.0138$, $d_4 = 0.0329$, $d_{5\mathrm{k}} = 0.0329$, $d_{5\mathrm{h}} = 0.0279$, $d_6 = 0.0250$, $d_{7\mathrm{k}} = 0.0250$, $d_{7\mathrm{h}} = 0.0619$, $d_8 = 0.0317$, $d_9 = 0.0368$, where subscripts a, k, and h designate the ankle, knee, and hip joints, respectively. The PCSA nominal values (in cm^2) were: $A_1 = 11.5$, $A_2 = 92.5$, $A_3 = 44.3$, $A_4 = 98.1$, $A_5 = 20.1$, $A_6 = 6.1$, $A_7 = 45.5$, $A_8 = 31.0$, $A_9 = 44.3$. (b) Twelve directions of exerted forces and corresponding joint moments. Joint moments (in N m) corresponding to a force of 63 N exerted isometrically on the force platform (indicated by the vertical rectangle) were taken from Table 5 of Wells and Evans (1987) (moment values are shown in the figure).

$$f_3 = d_{5\mathrm{h}}F_5 - d_{7\mathrm{h}}F_7 + d_8F_8 - d_9F_9 - M_3 = 0, \tag{2.3}$$

$$F_i \geqslant 0 \; (i = 1, 2, \ldots, 9), \tag{2.4}$$

where Z is the objective function, f_1, f_2, and f_3 are constraint functions, F_i the unknown force of the ith muscle (for specification of the indexes i see Fig. 1), A_i the PCSA of the ith muscle, d_i the moment arm of the

ith muscle and $d_i > 0$ by definition (moment arms of the two-joint muscles gastrocnemius (GA, $i = 3$), rectus femoris (RF, $i = 5$), and long head of biceps femoris (BFL, $i = 7$) with respect to the adjacent joints are denoted by subscripts a, k, and h, corresponding to the ankle, knee, and hip joints, respectively), M_1, M_2, and M_3 are the resultant moments at the ankle, knee, and hip joints, respectively. Resultant joint moments at the three joints, muscle moment arms and PCSAs are assumed known. The objective function (1) was proposed by Crowninshield and Brand (1981) based on the experimental relationship 'endurance time—muscle stress' in humans and has the meaning of muscle fatigue. According to Crowninshield and Brand, values of power n in that relationship range between 2.54 and 3.14. The values 2 and 3 were chosen for the power n in this study. The equality constraint Eqs. (2.1)–(2.3) require the optimal muscle forces to produce the known joint moments. The inequality constraint Eq. (2.4) requires muscles to produce force in pulling direction.

To obtain analytical relationships between the optimal forces and model parameters, the Lagrange multipliers method was employed (Challis and Kerwin, 1993; Dul et al., 1984; Raikova, 1992). For the above problem (1)–(2.3) the Lagrange function is: $L = Z - \lambda_1 f_1 - \lambda_2 f_2 - \lambda_3 f_3$ (where λ_1, λ_2 and λ_3 are unknown Lagrange multipliers associated with the constraint functions f_1, f_2, and f_3, respectively). The necessary conditions for the existence of an extremum of the Lagrange function, and hence of the objective function Z, are that all first partial derivatives of L with respect to the design variables F_i are zero at the optimal point:

$$\frac{\partial L}{\partial F_1} = \frac{nF_1^{n-1}}{A_1^n} - \lambda_1 d_1 = 0 \rightarrow F_1^{n-1} = \frac{\lambda_1 d_1 A_1^n}{n}, \tag{3.1}$$

$$\frac{\partial L}{\partial F_2} = \frac{nF_2^{n-1}}{A_2^n} + \lambda_1 d_2 = 0 \rightarrow F_2^{n-1} = -\frac{\lambda_1 d_2 A_2^n}{n}, \tag{3.2}$$

$$\frac{\partial L}{\partial F_3} = \frac{nF_3^{n-1}}{A_3^n} + \lambda_1 d_{3a} + \lambda_2 d_{3k} = 0 \rightarrow F_3^{n-1} = -\frac{(\lambda_1 d_{3a} + \lambda_2 d_{3k}) A_3^n}{n}, \tag{3.3}$$

$$\frac{\partial L}{\partial F_4} = \frac{nF_4^{n-1}}{A_4^n} - \lambda_2 d_4 = 0 \rightarrow F_4^{n-1} = \frac{\lambda_2 d_4 A_4^n}{n}, \tag{3.4}$$

$$\frac{\partial L}{\partial F_5} = \frac{nF_5^{n-1}}{A_5^n} - \lambda_2 d_{5k} - \lambda_3 d_{5h} = 0 \rightarrow F_5^{n-1} = \frac{(\lambda_2 d_{5k} + \lambda_3 d_{5h}) A_5^n}{n}, \tag{3.5}$$

$$\frac{\partial L}{\partial F_6} = \frac{nF_6^{n-1}}{A_6^n} + \lambda_2 d_6 = 0 \rightarrow F_6^{n-1} = -\frac{\lambda_2 d_6 A_6^n}{n}, \tag{3.6}$$

$$\frac{\partial L}{\partial F_7} = \frac{nF_7^{n-1}}{A_7^n} + \lambda_2 d_{7k} + \lambda_3 d_{7h} = 0 \rightarrow F_7^{n-1} = -\frac{(\lambda_2 d_{7k} + \lambda_3 d_{7h}) A_7^n}{n}, \tag{3.7}$$

$$\frac{\partial L}{\partial F_8} = \frac{nF_8^{n-1}}{A_8^n} - \lambda_3 d_8 = 0 \rightarrow F_8^{n-1} = \frac{\lambda_3 d_8 A_8^n}{n}, \tag{3.8}$$

$$\frac{\partial L}{\partial F_9} = \frac{nF_9^{n-1}}{A_9^n} + \lambda_3 d_9 = 0 \rightarrow F_9^{n-1} = -\frac{\lambda_3 d_9 A_9^n}{n}. \tag{3.9}$$

From Eqs. (3.1)–(3.9) and the requirements $F_i \geqslant 0$ (Eq. (2.4)), $d_i > 0$, $n > 0$, and $A_i > 0$, it is easy to see that some muscle forces in the optimal solution cannot have non-zero values without violating the above requirements. Some forces must be zero. Which one is zero depends on λ_j signs. The signs of λ_j ($j = 1, 2, 3$) are not known in general because they depend on current values of input (joint moments) and model parameters (muscle moment arms and PCSAs; see Eqs. (5.1)–(5.3) below). In general, there may be eight sign combinations of λ_j as demonstrated in Table 1, top row (trivial cases where some of λ_j equal zero are not considered here). The corresponding muscle active/silent state combinations (Table 1) can be easily derived from given λ_j signs, Eqs. (3.1)–(3.9), and the conditions $d_i > 0$, $A_i > 0$, $F_i \geqslant 0$ ($i = 1, 2, \ldots, 9$). The force combinations in Table 1 hold for $n > 1$.

Table 1
Possible sign combinations of λ_j ($j = 1, 2, 3$) and the corresponding muscle active/silent states for the optimal solutions of the problem (1)–(2.4) with $n > 1$[a]

	A	B	C	D	E	F	G	H
	$\lambda_1 > 0$ $\lambda_2 > 0$ $\lambda_3 > 0$	$\lambda_1 < 0$ $\lambda_2 < 0$ $\lambda_3 < 0$	$\lambda_1 > 0$ $\lambda_2 > 0$ $\lambda_3 < 0$	$\lambda_1 > 0$ $\lambda_2 < 0$ $\lambda_3 > 0$	$\lambda_1 < 0$ $\lambda_2 > 0$ $\lambda_3 > 0$	$\lambda_1 > 0$ $\lambda_2 < 0$ $\lambda_3 < 0$	$\lambda_1 < 0$ $\lambda_2 > 0$ $\lambda_3 < 0$	$\lambda_1 < 0$ $\lambda_2 < 0$ $\lambda_3 > 0$
F_1; TA	$\vee$	$\varnothing$	$\vee$	$\vee$	$\varnothing$	$\vee$	$\varnothing$	$\varnothing$
F_2; SO	$\varnothing$	$\vee$	$\varnothing$	$\varnothing$	$\vee$	$\varnothing$	$\vee$	$\vee$
F_3; GA	$\varnothing$	$\vee$	$\varnothing$	*	*	*	*	$\vee$
F_4; VA	$\vee$	$\varnothing$	$\vee$	$\varnothing$	$\vee$	$\varnothing$	$\vee$	$\varnothing$
F_5; RF	$\vee$	$\varnothing$	*	*	$\vee$	$\varnothing$	*	*
F_6; BFS	$\varnothing$	$\vee$	$\varnothing$	$\vee$	$\varnothing$	$\vee$	$\varnothing$	$\vee$
F_7; BFL	$\varnothing$	$\vee$	*	*	$\varnothing$	$\vee$	*	*
F_8; IL	$\vee$	$\varnothing$	$\varnothing$	$\vee$	$\vee$	$\varnothing$	$\varnothing$	$\vee$
F_9; GLM	$\varnothing$	$\vee$	$\vee$	$\varnothing$	$\varnothing$	$\vee$	$\vee$	$\varnothing$

[a] Muscle names are given in Fig. 1; λ_i are Lagrange multipliers; $\vee$ and $\varnothing$ indicate active and silent muscle states, respectively; * indicates that the corresponding two-joint muscle may be active or silent depending, in particular, on the relation between its moment arms at the two joints (see Eqs. (3.1)–(3.9)). For example in Column C, if $\lambda_2 d_{5k} + \lambda_3 d_{5h} \leqslant 0$, RF has silent state. The number of muscles with active state is between three (columns C, D and G) and six (columns D, G and H). One-joint antagonists (TA vs. SO, VA vs. BFS, and IL vs. GLM) cannot have active states simultaneously, but two-joint antagonists (RF vs. BFL; columns C, D, G and H) and one-joint muscles with their two-joint antagonists (e.g., TA vs. GA, columns D and F) can have active states simultaneously.

Consider $n = 2$. Substituting the expressions for F_i from Eqs. (3.1)–(3.9) in the joint moment equations $f_i(F_i) = 0$ (Eqs. (2.1)–(2.3)) the following system for λ_j is obtained:

$$c_{11}\lambda_1 + c_{12}\lambda_2 = 2M_1, \tag{4.1}$$

$$c_{12}\lambda_1 + c_{22}\lambda_2 + c_{23}\lambda_3 = 2M_2, \tag{4.2}$$

$$c_{23}\lambda_2 + c_{33}\lambda_3 = 2M_3, \tag{4.3}$$

where $c_{11} = d_1^2A_1^2 + d_2^2A_2^2 + d_{3a}^2A_3^2$; $c_{12} = d_{3a}d_{3k}A_3^2$; $c_{22} = d_{3k}^2A_3^2 + d_4^2A_4^2 + d_{5k}^2A_5^2 + d_6^2A_6^2 + d_{7k}^2A_7^2$; $c_{23} = d_{5k}d_{5h}A_5^2 + d_{7k}d_{7h}A_7^2$; $c_{33} = d_{5h}^2A_5^2 + d_{7h}^2A_7^2 + d_8^2A_8^2 + d_9^2A_9^2$. Solving system (4.1)–(4.3) for λ_j ($j = 1, 2, 3$) we obtain:

$$\lambda_1 = \frac{2(M_1c_{33}c_{22} - M_1c_{23}^2 - M_2c_{12}c_{33} + M_3c_{12}c_{23})}{c_{11}c_{22}c_{33} - c_{12}^2c_{33} - c_{23}^2c_{11}}, \tag{5.1}$$

$$\lambda_2 = \frac{2(-M_1c_{12}c_{33} + M_2c_{11}c_{33} - M_3c_{23}c_{11})}{c_{11}c_{22}c_{33} - c_{12}^2c_{33} - c_{23}^2c_{11}}, \tag{5.2}$$

$$\lambda_3 = \frac{2(M_1c_{12}c_{23} - M_2c_{11}c_{23} + M_3c_{11}c_{22} - M_3c_{12}^2)}{c_{11}c_{22}c_{33} - c_{12}^2c_{33} - c_{23}^2c_{11}}, \tag{5.3}$$

where it is supposed that $c_{11}c_{22}c_{33} - c_{12}^2c_{33} - c_{23}^2c_{11} \neq 0$. Thus, λ_j are expressed through known model and input parameters d_i, A_i and M_j. Combining the Eqs. (5.1)–(5.3) and (3.1)–(3.9) yields analytical expressions relating muscle forces F_i to d_i, A_i and M_j, which are given in detail in Prilutsky (2000, pp. 109–110). It must be emphasized that the above relationships satisfy only the necessary conditions for the existence of an extremum of the objective function Z, but the requirements for non-negativity of the muscle forces are not taken into account. Nevertheless, since the non-zero optimal muscle forces for the problem (1)–(2.4) must satisfy the Eqs. (3.1)–(3.9) and (5.1)–(5.3) (or the equations in Prilutsky, 2000), these equations show how the non-zero optimal forces depend on input and model parameters.

In the optimal solution some of the forces must be zero (see Table 1). Which of them are zero depends on the signs of λ_j, hence, on parameters d_i, A_i and M_j (see Eqs. (5.1)–(5.3)). The fact that some forces are zero in the optimal solution can be used to find the optimal solution from Eqs. (3.1)–(3.9) and (5.1)–(5.3) by setting values of moment arms for muscles with the silent state to zero. An example of the analytical solution of the problem (1)–(2.4) for $n = 2$ and $\lambda_j > 0$ (column A in Table 1) is given in Appendix. This solution is realized for the first five force directions (or joint moment combinations; Fig. 1b) and nominal values of the model parameters (see the caption of Fig. 1).

A similar approach was used in an algorithm to calculate the optimal forces for $n = 2$ and for arbitrary values of the model parameters. The algorithm was based on the fact that all sign combinations of λ_j shown in Table 1 (and, therefore, all muscle force combinations satisfying the Eqs. (3.1)–(3.9) and the inequality constraints (2.4)) could be obtained by setting consecutively all d_i (hence the moment of F_i) to zero. Values of λ_j were calculated from Eqs. (5.1)–(5.3) and the muscle forces from Eqs. (3.1)–(3.9). Solutions that either did not satisfy moment constraints (2.1)–(2.3) or in which at least one muscle force was found negative were rejected. From the remaining solutions, the one with the minimum value of the objective function was selected. Since the Hessian matrix of the Lagrange function is positive-definite for $n > 1$, the objective function (1) reaches its minimum, so the selected solution is the solution of the optimization problem (1)–(2.4) (see, for example, Bertsekas, 1996).

Pure analytical expressions relating optimal muscle forces F_i to joint moments and model parameters for the case $n = 3$ (Eq. (1)) were impossible to derive because the corresponding system for λ_j could not be solved analytically. For solving the optimization problem for $n = 3$ and for cross-validating the solutions obtained for $n = 2$, the routine *constr* of the optimization toolbox of MATLAB (the MathWorks Inc., Natick, MA, USA) was used.

To cross-validate the numerical solutions for $n = 3$, the same optimization problem was solved for nominal values of the model parameters and all combinations of joint moments (Fig. 1b) using a semi-analytical approach. All muscle force combinations corresponding to the eight sign variants of λ_j from Table 1 were considered separately. The constrained optimization problem was reduced to an unconstrained one by expressing some of the muscle forces from the moment equations and substituting those expressions in the objective function. All local minima of the obtained function, for which $F_i \geqslant 0$, were calculated and the muscle force combination with the lowest cost function value (the optimal solution) was selected. The detailed description of this algorithm is not presented here because of space limitation.

2.2. Sensitivity analysis

A sensitivity analysis of the optimal solution of the problem (1)–(2.4) was performed analytically for $n = 2$ and numerically for $n = 3$. Combinations of joint moments necessary for this analysis were taken from the study of Wells and Evans (1987). In their study, female subjects were laying on their left side with the right hip and knee angles at 105° and 90°, respectively, and their ankle angle at about 120° (Fig. 4 in Wells and Evans and Fig. 1a in this paper). The right leg was supported by suspending it with cords from the ceiling. The subjects were instructed to use their right leg to exert a force of 63 N in the horizontal plane in twelve different directions (Fig. 1b) using visual feedback on the magnitude and direction of the force. The force was measured by a force platform mounted vertically and connected to the subject's foot by a Nordic ski boot. The resultant moments at the ankle, knee, and hip joints were calculated (they are listed in Fig. 1b). Surface electromyographic activity (EMG) of major leg muscles was recorded, full-wave rectified, low-pass filtered at 6 Hz, averaged over 1 s, and expressed as a fraction of maximum EMG recorded in maximal isometric contractions at the same leg position. In the present study, EMG data from Wells and Evans' paper (their Fig. 6) were magnified and the EMG magnitude of each muscle was estimated for each force direction.

The sensitivity analysis of the present study was performed by calculating the optimal muscle forces for the joint moments given in Fig. 1B by changing only one parameter (muscle moment arm or PCSA of one muscle) while keeping the remaining parameters unchanged. The nominal values of the muscle moment arms (d_{oi}) were calculated from the joint angles using the equations reported by Prilutsky and Gregor (1997, their Table 2). The nominal values of PCSAs (A_{oi}) were taken from Table 1 of Prilutsky and Gregor (1997). These nominal values are listed in the captions of Fig. 1. Each moment arm was changed from 0 to $2.0d_{oi}$ with an increment of $0.02d_{oi}$. Each PCSA was changed from 0.05 to $2A_{oi}$ with an increment of $0.01A_{oi}$.

3. Results

It is evident from the analytical relationships between optimal muscle forces and model parameters obtained for $n = 2$ that each non-zero muscle force generally depends in a very complex non-linear way on moments at all three joints and moment arms and PCSAs of all nine muscles (see Eqs. (3.1)–(3.9) and (5.1)–(5.3), Appendix, or the formulas in Prilutsky, 2000). The number of non-zero optimal forces for the problem (1)–(2.4), with $n > 1$ and $\lambda_j \neq 0$, depends on the signs of λ_j (hence, on model parameters and joint moments), but cannot exceed six (Table 1, columns D, G, H) or be smaller than three (Table 1, columns C, D and G). It also follows from Table 1 that one-joint anatomical antagonists (i.e., tibialis anterior (TA) and soleus (SO), vastii (VA) and short head of biceps femoris (BFS), iliacus (IL) and gluteus maximum (GLM)) cannot produce non-zero forces simultaneously, whereas simultaneous force production by two-joint antagonists (RF and BFL, see Table 1 C, D, G and H) and by one-joint muscles and their two-joint anatomical antagonists (e.g., TA and GA, IL and BFL, etc., see Table 1 D, F and H) could be predicted for specific input and model parameters.

The results of the sensitivity analysis are summarized in Figs. 2–5. Variations in both muscle moment arms (Figs. 2, 4b and 5b) and PCSAs (Figs. 3, 4a and 5a) modified optimal forces calculated using the two investigated cost functions with generally more pronounced effects from the moment arms than PCSAs. For example, peak forces of TA, GA, VA, BFS, GLM, and RF increased or decreased several times when muscle moment arms were changed from their nominal values (Fig. 2; see also the influence of $d_{7\mathrm{h}}$ on the GLM force, Figs. 4b and 5b). Although the sensitivity of the optimal forces to changes in PCSAs was smaller, in general, than the sensitivity to moment arms (with one exception: RF, Figs. 2 and 3), the effects of the PCSAs variation was also rather large for some muscles (VA, RF, BFS, Fig. 3; see also the influence of A_7 and A_9 on muscle forces, Figs. 4a and 5a). Force of BFL predicted by both criteria was not very sensitive to values of PCSAs in the physiologically feasible range (Fig. 3).

Deviations of the model parameters from their nominal values also affected patterns of predicted forces (e.g., positions of their peaks and zero values with respect to the external force direction; Fig. 2: TA, directions 9–11; SO and GLM, directions 7–11; VA, directions 3–6 and 9–11; BFS, directions 9–12; IL, directions 8–11; see also Fig. 3). Model parameter variations could change the set and the number of muscles with non-zero forces in the optimal solution. For example, at force direction 11 (Fig. 4b) when $d_{7\mathrm{k}}$ varied from 0 to 3.75 cm, four different combinations of non-zero muscle forces constituted the optimal solution for the quadratic criterion: TA, GA, BFS, BFL, GLM if $d_{7\mathrm{k}} \in [0, 0.14]$ or $d_{7\mathrm{k}} \in [1.13, 1.38]$; TA, GA, BFS, BFL, IL if $d_{7\mathrm{k}} \in [0.15, 1.12]$; SO, GA, BFS, BFL, GLM if $d_{7\mathrm{k}} \in [1.39, 2.46]$; and SO, GA, VA, BFL, GLM if $d_{7\mathrm{k}} \in [2.47, 3.75]$. Similar results were obtained for $n = 3$ (Fig. 5b).

Criteria with powers $n = 2$ and $n = 3$ predicted similar optimal forces for nominal and changed model parameters (Fig. 2 vs. Fig. 3, Fig. 4a vs. Fig. 5a, Fig. 4b vs. Fig. 5b). Virtually, at all force directions (joint moment combinations) the criteria with $n = 2$ and $n = 3$ predicted the same muscles to have non-zero forces. Only at force direction 11, the muscle sets with active states obtained for nominal values of the model parameters were different between the two criteria: SO, GA, VA, BFL and GLM for $n = 2$ and SO, GA, BFS, BFL and GLM for $n = 3$.

The increase in PCSA of a given muscle, typically, led to an increase of the predicted force in the same muscle, whereas predicted forces of other muscles might increase, decrease or remain unchanged (Figs. 4a and 5a). The relationship between the moment arm of a given muscle and its predicted force was more complex. For example, increasing the TA moment arm caused its force to decrease at force direction 6, but to increase at force direction 7. Furthermore, increasing moment arms of a two-joint muscle at each joint it spans might cause opposite changes in its force: for example, at force direction 7, GA force decreased with increasing $d_{3\mathrm{a}}$, but increased with increasing $d_{3\mathrm{k}}$.

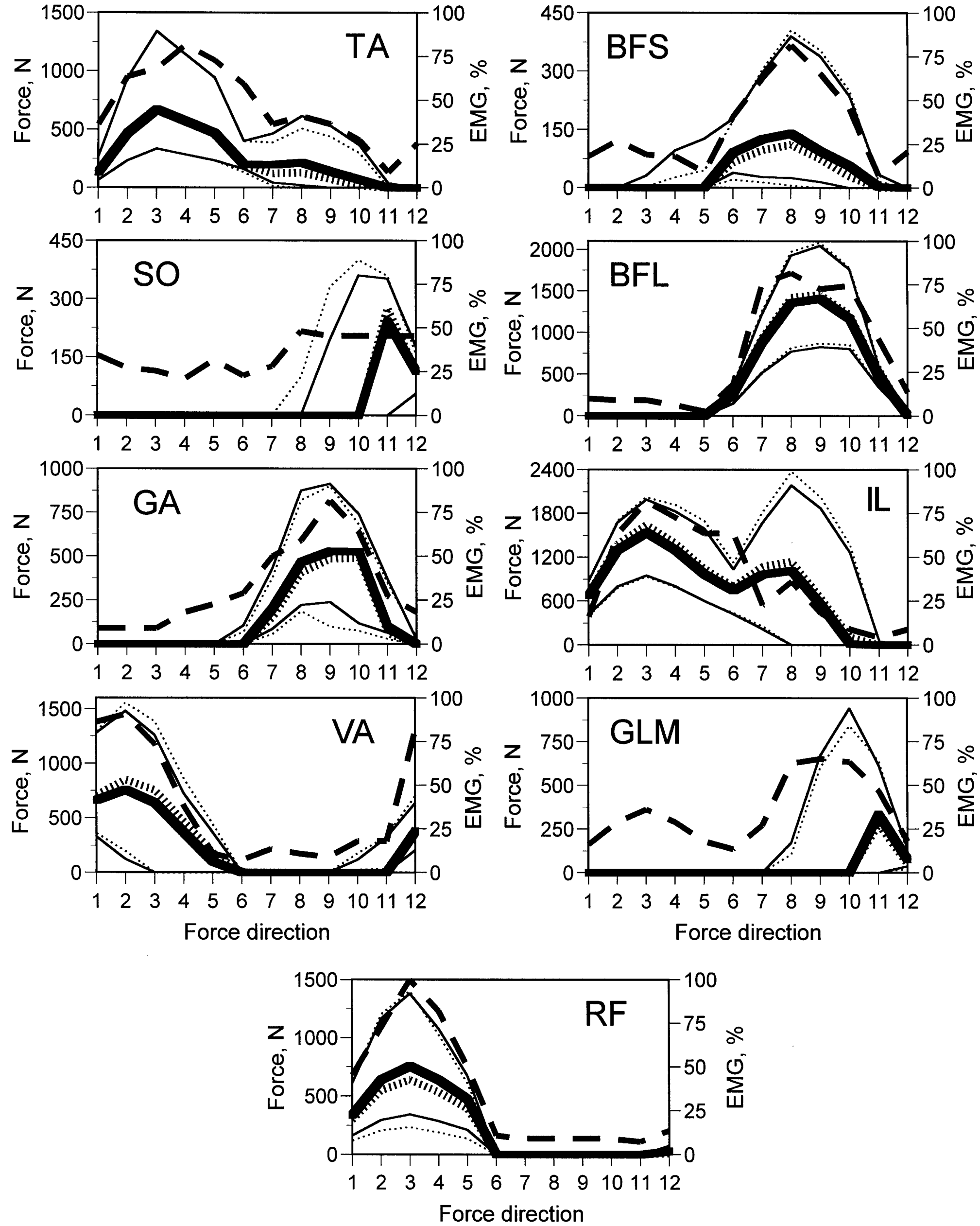


Fig. 2. Muscle forces predicted by minimizing quadratic (dotted lines) and cubic (continuous lines) objective functions (see Eq. (1)) for 12 force directions while changing moment arms. Thick lines correspond to forces obtained with nominal values of the muscle moment arms d_{oi}; thin lines correspond to maximal and minimal values of the predicted forces obtained through changing all moment arms consecutively from 0.05 to $2d_{oi}$ with a step of $0.05d_{oi}$; dashed lines are normalized EMG values estimated from Fig. 6 of Wells and Evans (1987). For muscle abbreviations see Fig. 1. Maximal and minimal values of the predicted forces of a given muscle were often predicted when the moment arm of another muscle was altered. Changing muscle moment arms within a physiologically feasible range ($0.05-2d_{oi}$) might alter muscle forces predicted by the two criteria by several times. Patterns of muscle forces as functions of force direction were less sensitive to variation in moment arms: the Pearson correlation coefficients calculated between nominal and maximal and also between nominal and minimal muscle forces were typically 0.9 and higher. However, variations in moment arms could change muscles' active/silent states and the number of non-zero forces in the optimal solution. Muscle forces corresponding to the quadratic and cubic criteria were similar.

The Pearson correlation coefficients calculated between estimated EMG and forces predicted using nominal values of model parameters ranged between 0.85 and 0.99 for both criteria and seven muscles. For two muscles, SO and GLM, the correlation coefficients were low.

4. Discussion

The aims of this study were (i) to attempt to find analytical relationships between model parameters and muscle forces minimizing the sum of muscle stresses squared and the sum of muscle stresses cubed for a

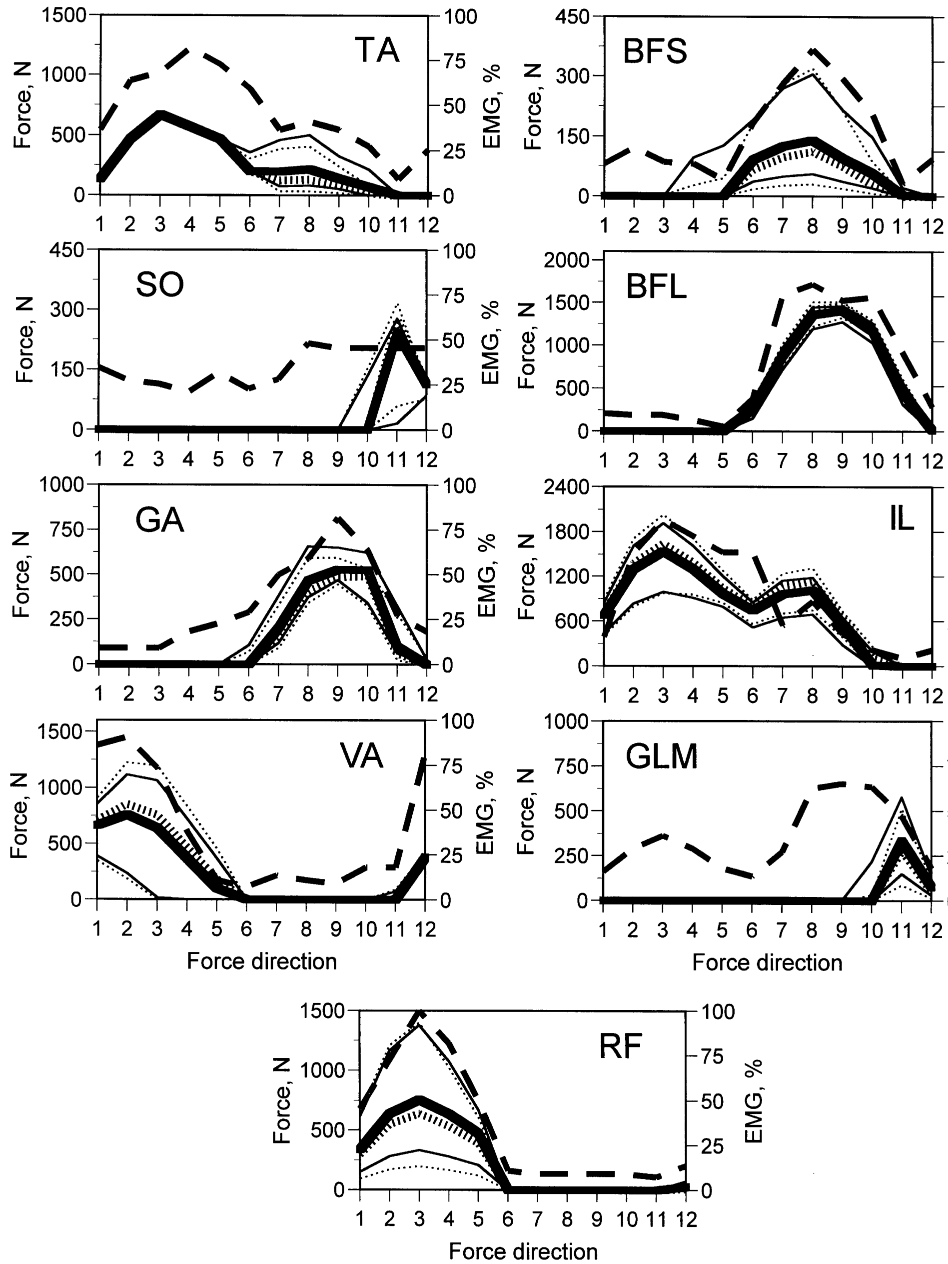


Fig. 3. Muscle forces predicted by minimizing quadratic (dotted lines) and cubic (continuous lines) objective functions (see Eq. (1)) for 12 force directions while changing PCSAs. Thick lines correspond to forces obtained with nominal values of PCSAs A_{oi}; thin lines correspond to maximal and minimal values of the predicted forces obtained through changing all PCSAs consecutively from 0.05 to $2A_{oi}$ with a step of $0.05A_{oi}$; dashed lines are normalized EMG values estimated from Fig. 6 of Wells and Evans (1987). For muscle abbreviations, see Fig. 1. Maximal and minimal values of the predicted forces of a given muscle were often predicted when PCSA of another muscle was altered. Sensitivity of the predicted muscle force magnitude to variations in PCSAs within a physiologically feasible range ($0.05-2A_{oi}$) was smaller compared to similar variations in moment arms (Fig. 2). However, variations in PCSAs also could change muscles' active/silent states and the number of non-zero forces in the optimal solution.

realistic 3DOF model of the human leg, and (ii) to investigate systematically the sensitivity of the optimal solution to model parameters (muscle moment arms and PCSAs). It was possible to obtain the sought analytical relationships only for the quadratic optimization criterion. They were much more complex than those for 1DOF systems (Challis and Kerwin, 1993; Dul et al., 1984; Raikova, 1996; Zatsiorsky et al., 1998). In particular, the non-zero optimal force of each muscle was generally found to be a very complex non-linear function of moments at all three joints and moment arms and PCSAs of all muscles. As a result, variation in one model parameter (moment arm or PCSA of one muscle) within a physiologically feasible range can

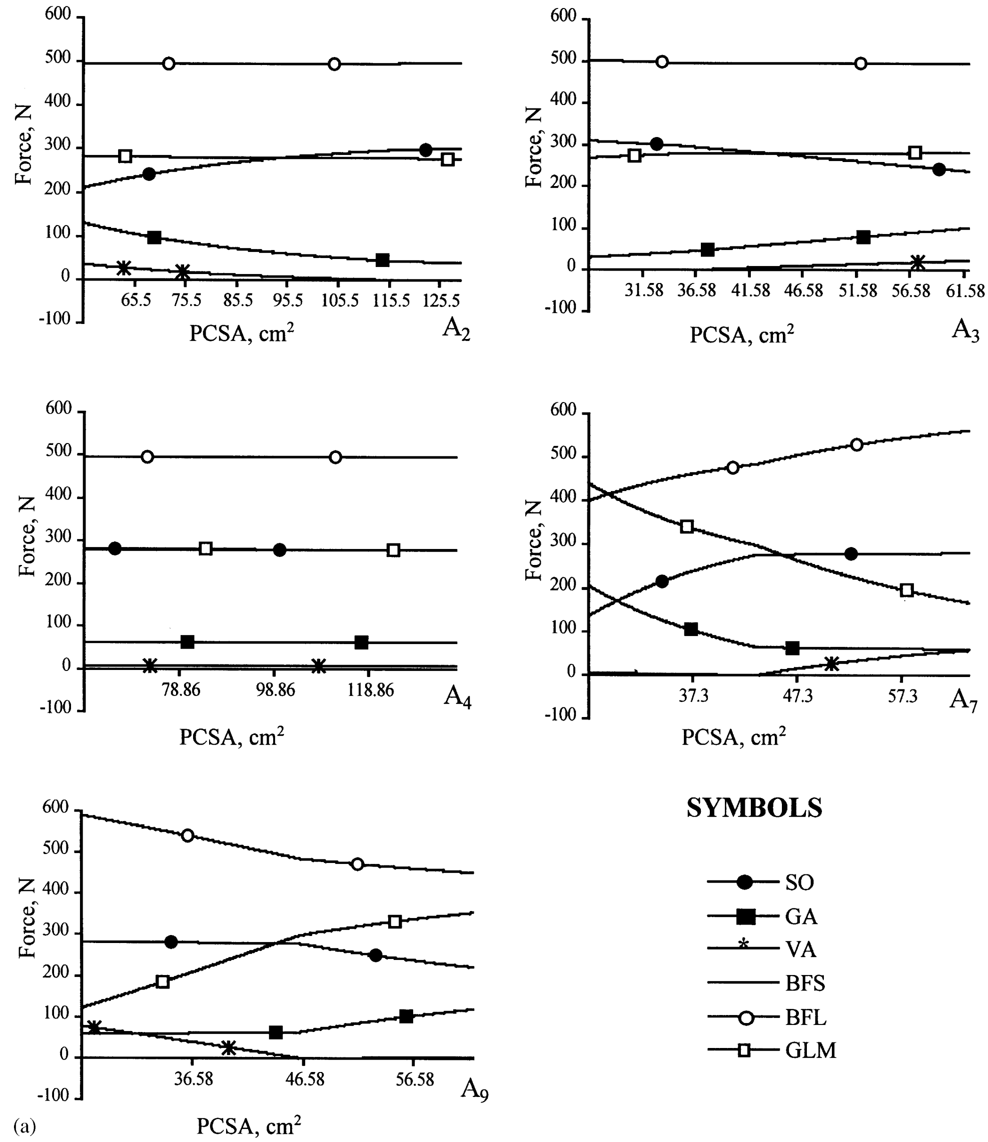


Fig. 4. Predicted muscle forces as functions of PCSAs (a) and moment arms (b) of muscles that influence the optimal solution: quadratic criterion. The plots for the remaining muscles (whose PCSAs and moment arms do not change the optimal solution) are not shown. Force direction 11; the corresponding joint moments are $M_1 = -15\,\mathrm{N\,m}$, $M_2 = -13\,\mathrm{N\,m}$, and $M_3 = -41\,\mathrm{N\,m}$. In each panel, only forces that have non-zero values within the range of the changing parameter are shown. (a) Predicted muscle forces as functions of PCSAs (A_i). Each A_i was changed within the range of $0.4A_{oi}-1.4A_{oi}$ with a step of $0.01A_{oi}$, while all other parameters had their nominal values. (b) Predicted muscle forces as functions of moment arms. Each d_i was changed from 0 to $1.5d_{oi}$ with a step of $0.02d_{oi}$. Both force-PCSA and force-moment arm relationships are non-linear in general. Variation in a parameter of one muscle could affect optimal forces of several other muscles and could change the number and set of muscles with active/silent states. Typically, the increase of muscle PCSA led to an increase in the force of the same muscle.

profoundly change not only force magnitude of this and other muscles (Figs. 2–4) but also the number of active muscles in the optimal solution and the set of muscles with active/silent states (Fig. 4). Similar results were also obtained numerically for $n = 3$ in the present study (Figs. 2, 3 and 5) and in Nussbaum et al. (1995) who investigated a spine model. Several conclusions, which were derived analytically for criterion (1) using 1DOF models (Hughes and Chaffin, 1988) or simplified multi-degree-of-freedom models (where all moment arms and PCSAs were assumed equal; Herzog and Binding, 1992, 1993), were confirmed in the present study for a more complex model: one-joint anatomical antagonists cannot produce force simultaneously; two-joint anatomical antagonists can have non-zero predicted forces simultaneously; and one-joint muscles and their two-joint antagonists can also have non-zero forces at the same time (see Table 1).

Muscle force sensitivity to moment arm variations was very substantial for all muscles and both criteria

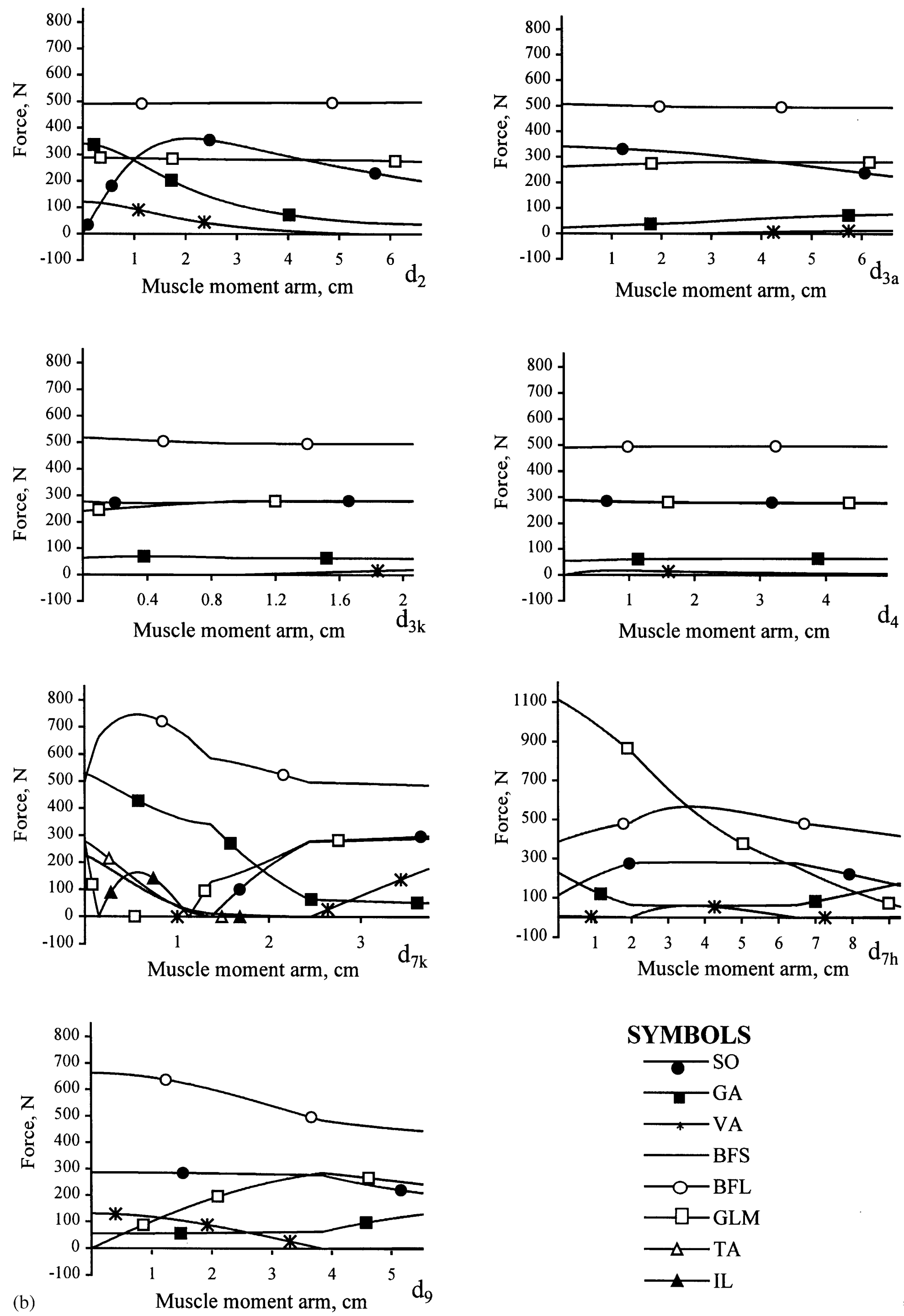


Fig. 4. (*Continued*).

—the difference between the lowest and highest predicted forces were several fold (Figs. 2, 4b and 5b). The sensitivity to PCSA variations was much smaller compared to the effect of variations in moment arms especially for TA, GA, and BFL, but was also substantial (Figs. 3, 4a and 5a). According to van Bolhuis and Gielen (1999), the optimal solutions of the problem (1)–(2.4) with $n = 2$ and 3 were not very sensitive to 25%-variations in moment arms and PCSA of the human arm muscles. These authors, however, did not compare magnitudes of the muscle forces corresponding to different model parameters, but rather compared force patterns (i.e., the correlation coefficients between the predicted muscle forces and the measured EMGs). Likewise, in the present study, force patterns were not substantially affected by changing model

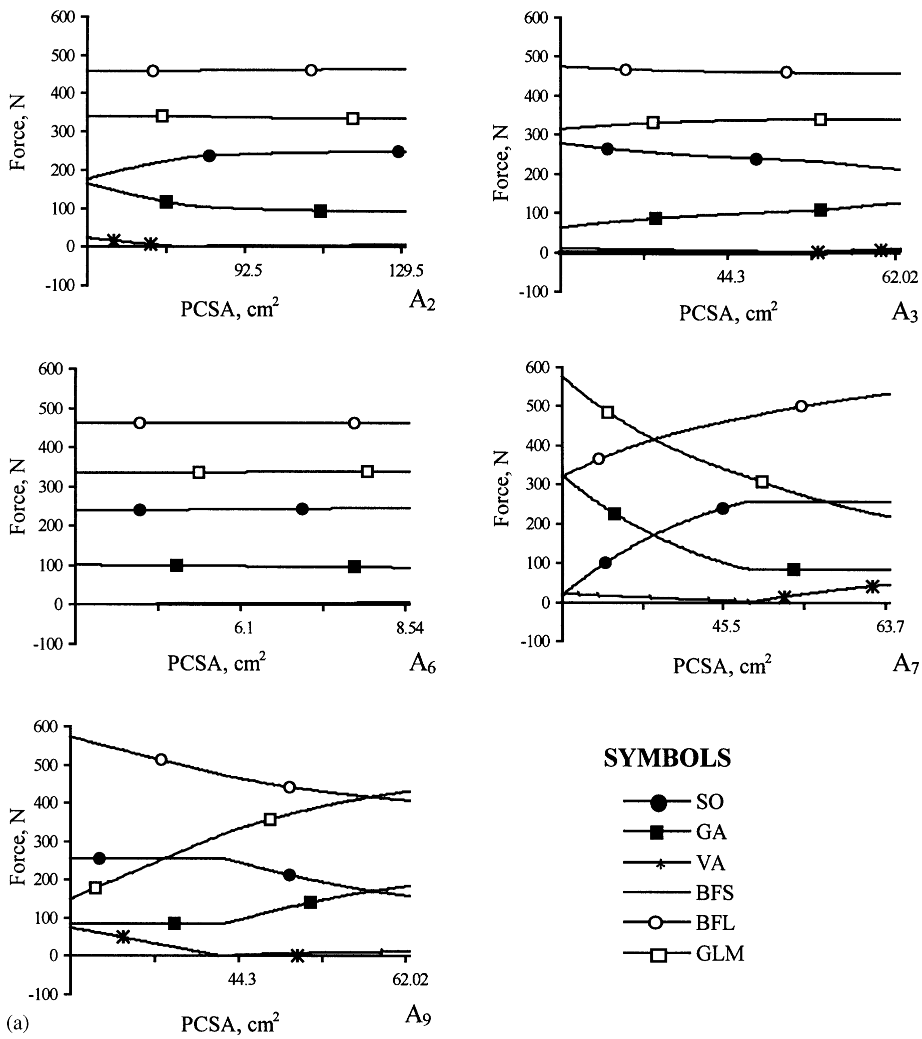


Fig. 5. Predicted muscle forces as functions of PCSAs (a) and moment arms (b) of muscles that influence the optimal solution: cubic criterion. The plots for the remaining muscles (whose PCSAs and moment arms do not change the optimal solution) are not shown. Force direction 11; the corresponding joint moments are $M_1 = -15\,\mathrm{N\,m}$, $M_2 = -13\,\mathrm{N\,m}$, and $M_3 = -41\,\mathrm{N\,m}$. In each panel, only forces that have non-zero values within the range of the changing parameter are shown. (a) Predicted muscle forces as functions of PCSAs (A_i). Each A_i was changed within the range of $0.4A_{oi}-1.4A_{oi}$ with a step of $0.01A_{oi}$, while all other parameters had their nominal values. (b) Predicted muscle forces as functions of moment arms. Each d_i was changed from 0 to $1.5d_{oi}$ with a step of $0.02d_{oi}$. Patterns of force-PCSA and force-moment arm relationships obtained for $n = 3$ were similar in general to those for $n = 2$ (Fig. 4), however, force magnitudes differed. Also, the set of muscles, whose parameters influenced the optimal solution, was changed from {SO, GA, VA, BFL, GLM} at $n = 2$ to {SO, GA, BFS, BFL, GLM} at $n = 3$.

parameters between 5% and 200% of their nominal values (Figs. 2 and 3): the correlation coefficients calculated, for example, between the maximum and nominal forces typically exceeded 0.9 for all muscles except GLM (Fig. 2). Comparable results were reported by Brand et al. (1986) who employed criterion (1) with $n = 3$. They found qualitatively similar muscle force patterns in walking for three sets of PCSA values which differed for some muscles by a factor of 10 or more. According to their results, the magnitude of the corresponding predicted forces also varied substantially—up to eight times. Note that an increase of PCSA of a given muscle typically leads to force increase of this muscle (Figs. 4a and 5a; see also Raikova, 2000b). Thus, it is important to use accurate estimates of model parameters if one is interested in accurate quantitative predictions of muscle force magnitudes. Qualitative pattern comparisons between optimum forces and EMG using correlation coefficients seem less affected by uncertainties in model parameters.

Although optimal forces corresponding to the criteria with powers $n = 2$ and $n = 3$ were very similar in terms of both magnitudes and patterns in general (Figs. 2 and 3, Fig. 4 vs. Fig. 5), a small difference in power (2 vs. 3) can change the set of active muscles in the optimum solution. For example, at force direction 11 and nominal

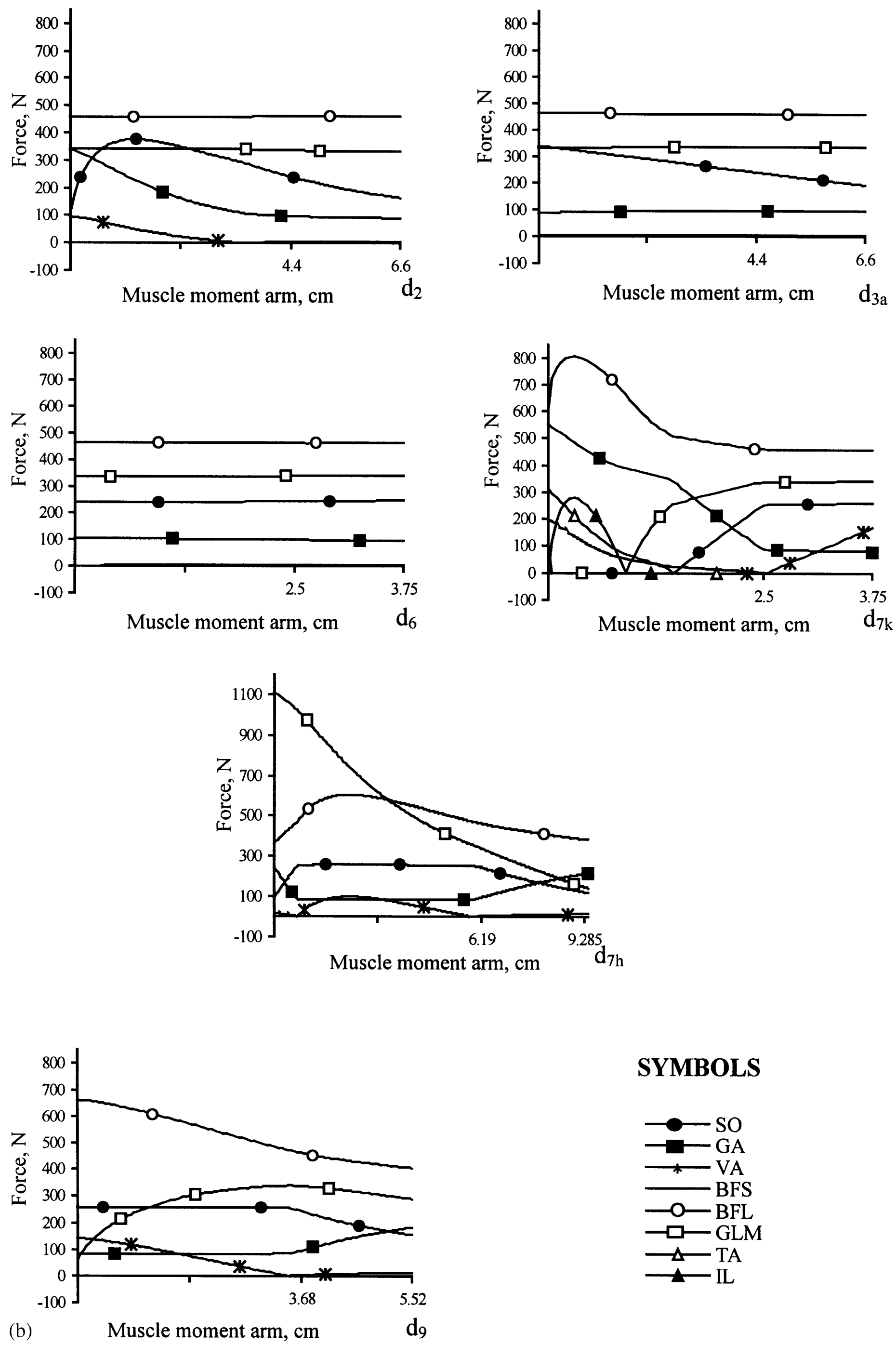


Fig. 5. (*Continued*).

values of model parameters, the optimal solutions were (in N): for $n = 2$, $F_1 = F_5 = F_6 = F_8 = 0$, $F_2 = 277.9$, $F_3 = 63.04$, $F_4 = 8.1$, $F_7 = 495.8$, and $F_9 = 280.2$; for $n = 3$, $F_1 = F_4 = F_5 = F_8 = 0$, $F_2 = 242.7$, $F_3 = 98.2$, $F_6 = 3.9$, $F_7 = 461.9$, and $F_9 = 337.2$. The relatively small difference between solutions obtained with $n = 2$ and $n = 3$ is more consistent with the results of Crowninshield and Brand (1981) than with those of van Bolhuis and Gielen (1999). The latter authors reported a substantial difference in force patterns calculated by the two criteria, although correlation coefficients between the predicted forces and the EMGs were similar for both criteria.

The results of the present study have important implications for validating muscle forces predicted by the optimization-based models. The quantitative validation of predicted muscle forces by comparing them with the measured EMG or even forces (Arndt et al., 1998;

Schuind et al., 1992) is warranted only if model parameters, especially, moment arms of each muscle, are accurately estimated. While qualitative validations using correlation coefficients are less sensitive to variations in model parameters in general (see the above discussion), some muscles can be predicted to have distinctly different force patterns at specific combinations of joint moments and different values of model parameters (see, for example, Fig. 2, SO and GLM). Therefore, the correlation between predicted forces and the measured EMG or force patterns may be affected by model parameters. Both the quantitative and qualitative validations can also be affected by artifacts of EMG and force recordings, by muscle physiological and mechanical properties, by a motor task, and the level of skill acquisition.

In conclusion, this study demonstrated using a rather realistic 3DOF model that, generally, the non-zero optimal force of each muscle depends, in a very complex non-linear way, on moments at all joints and moment arms and PCSAs of all muscles; changes in model parameters of one muscle can change predicted forces in the same and other muscles by several times and can also change the number of non-zero forces in the optimal solution and the set of muscles with active states; predicted muscle forces are more sensitive to changes in moment arms than to changes in PCSA; and changes in model parameters have a much stronger effect on the magnitude of predicted forces than on their patterns.

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Appendix. Optimal solution of the problem (1)–(2.4) for $n=2$ and λ_j sign combinations in column A of Table 1

Since $\lambda_j > 0$ ($j = 1, 2, 3$; Table 1, column A), then $F_2 = F_3 = F_6 = F_7 = F_9 = 0$.

Substituting $d_2 = d_{3a} = d_{3k} = d_6 = d_{7k} = d_{7h} = d_9 = 0$ in the Eqs. (5.1)–(5.3) and (3.1)–(3.9), the following expressions are obtained:

$$c_{11} = d_1^2 A_1^2,\ c_{12} = 0,\ c_{22} = d_4^2 A_4^2 + d_{5k}^2 A_5^2,$$

$$c_{23} = d_{5k} d_{5h} A_5^2,\ c_{33} = d_{5h}^2 A_5^2 + d_8^2 A_8^2,$$

$$\lambda_1 = \frac{2M_1}{d_1^2 A_1^2},$$

$$\lambda_2 = \frac{2[M_2(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - M_3 d_{5k} d_{5h} A_5^2]}{(d_4^2 A_4^2 + d_{5k}^2 A_5^2)(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - d_{5k}^2 d_{5h}^2 A_5^4},$$

$$\lambda_3 = \frac{2[M_3(d_{5k}^2 A_5^2 + d_4^2 A_4^2) - M_2 d_{5k} d_{5h} A_5^2]}{(d_4^2 A_4^2 + d_{5k}^2 A_5^2)(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - d_{5k}^2 d_{5h}^2 A_5^4},$$

$$F_1 = \frac{M_1}{d_1},$$

$$F_4 = d_4 A_4^2 \frac{M_2(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - M_3 d_{5k} d_{5h} A_5^2}{(d_4^2 A_4^2 + d_{5k}^2 A_5^2)(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - d_{5k}^2 d_{5h}^2 A_5^4},$$

$$F_5 = A_5^2 \frac{M_2 d_{5k} d_8^2 A_8^2 + M_3 d_{5h} d_4^2 A_4^2}{(d_4^2 A_4^2 + d_{5k}^2 A_5^2)(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - d_{5k}^2 d_{5h}^2 A_5^4},$$

$$F_8 = d_8 A_8^2 \frac{M_3(d_{5k}^2 A_5^2 + d_4^2 A_4^2) - M_2 d_{5k} d_{5h} A_5^2}{(d_4^2 A_4^2 + d_{5k}^2 A_5^2)(d_{5h}^2 A_5^2 + d_8^2 A_8^2) - d_{5k}^2 d_{5h}^2 A_5^4}.$$

This solution is realized for the first five force directions (or combinations of joint moments; see Fig. 1b) and nominal values of the model parameters (d_i [m] and A_i [cm^2], see the caption of Fig. 1). In particular, if $M_1 = 4$ N m, $M_2 = 33$ N m, and $M_3 = 31$ N m (direction 1, Fig. 1b), then $c_{11} = 0.1174$, $c_{12} = 0$, $c_{22} = 10.85$, $c_{23} = 0.3709$, $c_{33} = 1.2802$, $\lambda_1 = 68.12$, $\lambda_2 = 4.47$, $\lambda_3 = 47.14$, $F_2 = F_3 = F_6 = F_7 = F_9 = 0, F_1 = 134.2$, $F_4 = 707.7$, $F_5 = 295.4$, $F_8 = 718.0$, $Z = 940.6$ (Z is the objective function).

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