New Results on Extended Intuitionistic Fuzzy Index Matrices

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Introduction

In 1983, the author introduced the concept of an Intuitioniostic Fuzzy Set (IFS) as an extension of Zadeh's fuzzy sets. In the next year, he introduced the concept of an Index Matrix (IM) as an extension of the concept of a standard matrix. On the basis of both concepts, the objects Intuitionistic Fuzzy IMs (IFIMs) and Extended IFIMs (EIFIMs) appeared and all results over the different types of IMs, obtained up to 2014, were collected in a book of Springer.
Here, we discuss shortly some basic concepts related to intuitionistic fuzziness, IMs and EIFIMs and introduce new and specific operations, relations and operators over EIFIMS.

Short remarks on intuitionistic fuzziness

We define an IFS A in a fixed universe E as an object of the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For the needs of some definitions in Section 4, we define for the IFS A:

$$\mathcal{C}(A) = \{\langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y)\rangle | x \in E\},$$

$$\mathcal{I}(A) = \{\langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y)\rangle | x \in E\}.$$

Intuitionistic Fuzzy Pair (IFP) is an object with the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$. Its components (a and b) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc.

Let us have two IFPs $y = \langle p, q \rangle$ and $z = \langle r, s \rangle$.
The following relations have been defined in [**?**, **?**]:

$$\begin{array}{lll} y < z & \text{iff} & p < r \text{ and } q > s \\ y \leq z & \text{iff} & p \leq r \text{ and } q \geq s \\ y = z & \text{iff} & p = r \text{ and } q = z \\ y \geq z & \text{iff} & p \geq r \text{ and } q \leq s \\ y > z & \text{iff} & p > r \text{ and } q < s \end{array}$$

There are a lot of operations, defined over IFPs. For example:

$$\begin{aligned}
\neg y &= \langle q, p\rangle \\
y \wedge z &= \langle \min(p, r), \max(q, s)\rangle \\
y \vee z &= \langle \max(p, r)), \min(q, s)\rangle \\
y + z &= \langle p + r - p.r, q.s\rangle \\
y.z &= \langle p.r, q + s - q.s\rangle \\
y@z &= \langle \tfrac{p+r}{2}, \tfrac{q+s}{2}\rangle.
\end{aligned}$$

Now, there are definitions of more than 200 different implications and more than 50 different negations and the biggest part of them are non-classical ones. The classical negation is the one defined above.

Remarks on IMs and EIFIMs

Let $\mathcal{I}$ be a fixed set of indices and $\mathcal{R}$ be the set of real numbers. Let operations $\circ, * : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be fixed. For example, $\circ, * \in \{+, -, \times, :, \max, \min\}$, or others.
Let the sets K and L satisfy the condition: $K, L \subset \mathcal{I}$. An IM with real number elements is the object:

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where

$$K = \{k_1, k_2, ..., k_m\} \text{ and } L = \{l_1, l_2, ..., l_n\},$$

and for $1 \leq i \leq m$, and for $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

When set $\mathcal{R}$ is changed with:

- set $\{0, 1\}$, we obtain a particular case of an IM with elements being numbers 0 and 1;
- set of propositions or predicates, we obtain a logical IM;
- set of IFPs, we obtain an IFIM, etc.

Over IMs a lot of operations are defined. Below, we will mention some of them for the case of EIFIMs.

Now, for above sets K and L, the EIFIM is defined by:

$$[K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}] \equiv$$

$$\begin{array}{c|ccccc}
 & \langle l_1, \alpha_1^l, \beta_1^l \rangle & \dots & \langle l_j, \alpha_j^l, \beta_j^l \rangle & \dots & \langle l_n, \alpha_n^l, \beta_n^l \rangle \\
\hline
\langle k_1, \alpha_1^k, \beta_1^k \rangle & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle & \dots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j} \rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle \\
\vdots & \vdots & \dots & \vdots & \dots & \vdots \\
\langle k_i, \alpha_i^k, \beta_i^k \rangle & \langle \mu_{k_i,l_1}, \nu_{k_i,l_1} \rangle & \dots & \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle & \dots & \langle \mu_{k_i,l_n}, \nu_{k_i,l_n} \rangle \\
\vdots & \vdots & \dots & \vdots & \dots & \vdots \\
\langle k_m, \alpha_m^k, \beta_m^k \rangle & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle & \dots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j} \rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle
\end{array},$$

where for every $1 \le i \le m, 1 \le j \le n$:

$$\mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \in [0,1],$$

$$\alpha_i^k, \beta_i^k, \alpha_i^k + \beta_i^k \in [0,1],$$

$$\alpha_j^l, \beta_j^l, \alpha_j^l + \beta_j^l \in [0,1]$$

and here and below,

$$K^* = \{\langle k_i, \alpha_i^k, \beta_i^k \rangle | k_i \in K\} = \{\langle k_i, \alpha_i^k, \beta_i^k \rangle | 1 \le i \le m\},$$

$$L^* = \{\langle l_j, \alpha_j^l, \beta_j^l \rangle | l_j \in L\} = \{\langle l_j, \alpha_j^l, \beta_j^l \rangle | 1 \le j \le n\}.$$

Obviously, K^* and L^* are IFSs.
For two EIFIMs different operations are defined.

New operations, relations and operators over EIFIMs

First, we will extend the definitions of the two basic operations over EIFIMsso that the existing operations be particular cases.
Let everywhere below $\circ, * \in \{\vee, \wedge\}$ and let when $\circ$ or $*$ be $\vee$, then $\circ_1$ or $*_1$ be max and $\circ_2$ or $*_2$ be min; and when $\circ$ or $*$ be $\wedge$, then $\circ_1$ or $*_1$ be min and $\circ_2$ or $*_2$ be max. Let $e_\vee = \langle 0, 1\rangle$ and $e_\wedge = \langle 1, 0\rangle$.
For the EIFIMs
$A = [K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle\}], B = [P^*, Q^*, \{\langle \rho_{p_r,q_s}, \sigma_{p_r,q_s}\rangle\}]$,
operations that are analogous to the standard matrix operations of addition and multiplication are defined, as well as other specific ones.

Addition

$$A \oplus_{\circ}^{*} B = [T^*, V^*, \{\langle \varphi_{t_u, v_w}, \psi_{t_u, v_w} \rangle\}],$$

where

$$T^* = K^* \cup P^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K \cup P\},$$

$$V^* = L^* \cup Q^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in L \cup Q\},$$

$$\alpha_u^t = \begin{cases} \alpha_i^k, & \text{if } t_u \in K - P \\ \alpha_r^p, & \text{if } t_u \in P - K \\ *_1(\alpha_i^k, \alpha_r^p), & \text{if } t_u \in K \cap P \end{cases},$$

$$\beta_w^v = \begin{cases} \beta_j^l, & \text{if } v_w \in L - Q \\ \beta_s^q, & \text{if } t_w \in Q - L \\ *_2(\beta_j^l, \beta_s^q), & \text{if } t_w \in L \cap Q \end{cases},$$

and

$$
\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =
\begin{cases}
\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \\
& \quad \text{and } v_w = l_j \in L - Q \\
& \quad \text{or } t_u = k_i \in K - P \\
& \quad \text{and } v_w = l_j \in L; \\[2ex]
\langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \\
& \quad \text{and } v_w = q_s \in Q - L \\
& \quad \text{or } t_u = p_r \in P - K \\
& \quad \text{and } v_w = q_s \in Q; \\[2ex]
\langle \circ_1(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\
\quad \circ_2(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \quad \text{and } v_w = l_j = q_s \in L \cap Q \\[2ex]
e_\circ, & \text{otherwise}
\end{cases}
$$

Termwise multiplication

$$A \otimes_{\circ}^{*} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$T^* = K^* \cap P^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K \cap P\},$$

$$V^* = L^* \cap Q^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in L \cap Q\},$$

$$\alpha_u^t = *_1(\alpha_i^k, \alpha_r^p), \text{ for } t_u = k_i = p_r \in K \cap P,$$

$$\beta_w^v = *_2(\beta_j^l, \beta_s^q), \text{ for } v_w = l_j = q_s \in L \cap Q$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \circ_1(\mu_{k_i,l_j}, \rho_{p_r,q_s}), \circ_2(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle.$$

Multiplication

$$A \odot_{(\circ,\bullet)} B = [T^*, V^*, \langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle \}],$$

where

$$T^* = (K \cup (P - L))^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K \cup (P - L)\},$$

$$V^* = (Q \cup (L - P))^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in Q \cup (L - P)\},$$

$$\alpha_u^t = \begin{cases} \alpha_i^k, & \text{if } t_u = k_i \in K \\ \\ \alpha_r^p, & \text{if } t_u = p_r \in P - L \end{cases},$$

$$\beta_w^v = \begin{cases} \beta_j^l, & \text{if } v_w = l_j \in L - P \\ \\ \beta_s^q, & \text{if } t_w = q_s \in Q \end{cases},$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$= \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = l_j \in L - P - Q \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P - L - K \\ & \quad \text{and } v_w = q_s \in Q \\ \Big\langle \underset{l_j = p_r \in L \cap P}{\circ_1} \left(\bullet_1 (\mu_{k_i,l_j}, \rho_{p_r,q_s}) \right), & \text{if } t_u = k_i \in K \\ \quad \underset{l_j = p_r \in L \cap P}{\circ_2} \left(\bullet_2 (\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \right) \Big\rangle, & \quad \text{and } v_w = q_s \in Q \\ e_\circ, & \text{otherwise} \end{cases}$$

For operations Addition and Termwise multiplication, when operations $\circ$ and $*$ coincide, we obtain the definitions of these two operations in the standard form, but obviously, the present definitions are extensions of the older ones, because here we can modify independently the IM values and IM indices which in the older case is impossible.

Operations “reduction” and “projection” over IMs are the same for the EIFIMs.

Second, let the two EIFIMs A and B be given. Now, we will introduce the following definitions where $\subset$ and $\subseteq$ denote the set-theoretical relations *"strong inclusion"* and *"weak inclusion"*.
The strict relation "inclusion about dimension" is

$$A \subset_d B \text{ iff } (((K^* \subset P^*) \,\&\, (L^* \subset Q^*)) \vee ((K^* \subseteq P^*) \,\&\, (L^* \subset Q^*))$$

$$\vee((K^* \subset P^*) \,\&\, (L^* \subseteq Q^*))) \,\&\, (\forall k \in K)(\forall l \in L)(\langle a_{k,l}, b_{k,l} \rangle = \langle c_{k,l}, d_{k,l} \rangle).$$

The non-strict relation "inclusion about dimension" is

$$A \subseteq_d B \text{ iff } (K^* \subseteq P^*) \,\&\, (L^* \subseteq Q^*) \,\&\, (\forall k \in K)(\forall l \in L)$$

$$(\langle a_{k,l}, b_{k,l} \rangle = \langle c_{k,l}, d_{k,l} \rangle).$$

The strict relation "inclusion about element values" is

$$A \subset_v B \text{ iff } (K^* = P^*) \,\&\, (L^* = Q^*) \,\&\, (\forall k \in K)(\forall l \in L)$$

$$(\langle a_{k,l}, b_{k,l} \rangle < \langle c_{k,l}, d_{k,l} \rangle).$$

The non-strict relation "inclusion about element values" is

$$A \subseteq_v B \text{ iff } (K^* = P^*) \ \& \ (L^* = Q^*) \ \& \ (\forall k \in K)(\forall l \in L)$$

$$(\langle a_{k,l}, b_{k,l} \rangle \leq \langle c_{k,l}, d_{k,l} \rangle).$$

The strict relation "inclusion about first index values" is

$$A \subset_{i,f} B \text{ iff } (K^* = P^*) \ \& \ (\forall k_i = p_q \in K)((\alpha_i^k < \alpha_q^p) \ \& \ (\beta_i^k > \beta_q^p)).$$

The non-strict relation "inclusion about first index values" is

$$A \subseteq_{i,f} B \text{ iff } (K^* = P^*) \ \& \ (\forall k_i = p_q \in K)((\alpha_i^k \leq \alpha_q^p) \ \& \ (\beta_i^k \geq \beta_q^p).$$

The strict relation "inclusion about second index values" is

$$A \subset_{i,s} B \text{ iff } (L^* = Q^*) \ \& \ (\forall l_j = r_s \in L)((\alpha_j^l < \alpha_a^r) \ \& \ (\beta_j^l > \beta_s^r)).$$

The non-strict relation "inclusion about first index values" is

$$A \subseteq_{i,s} B \text{ iff } (L^* = Q^*) \ \& \ (\forall l_j = r_s \in L)((\alpha_j^l \leq \alpha_a^r) \ \& \ (\beta_j^l \geq \beta_s^r)).$$

Third, by analogy with the operation "substitution" defined over standard IMs, we introduce the same operation over EIFIMs. The new operation is specific only for this type of IMs.

Let the EIFIM A be given and let below $\perp$ denote a lack of components.

A local substitution of an index degree of the EIFIM is defined for the index $k \in K$ or $l \in L$, respectively, by

$$\left[\frac{\langle k,\gamma^k,\delta^k\rangle}{\langle k,\alpha^k,\beta^k\rangle};\bot\right]A$$

$$=\left[(K^*-\{\langle k,\alpha^k,\beta^k\rangle\})\cup\{\langle k,\gamma^k,\delta^k\rangle\},L,\{\langle a_{k,l},\mu_{k_i,l_j},\nu_{k_i,l_j}\rangle\}\right],$$

$$\left[\bot;\frac{\langle l,\gamma^l,\delta^l\rangle}{\langle l,\alpha^l,\beta^l\rangle}\right]A$$

$$=\left[K^*,(L^*-\{\langle l,\alpha^l,\beta^l\rangle\})\cup\{\langle l,\alpha^l,\beta^l\rangle\},L,\{\langle a_{l,l},\mu_{l_i,l_j},\nu_{l_i,l_j}\rangle\}\right],$$

$$\begin{aligned}\left[\frac{\langle k,\gamma^k,\delta^k\rangle}{\langle k,\alpha^k,\beta^k\rangle};\frac{\langle l,\gamma^l,\delta^l\rangle}{\langle l,\alpha^l,\beta^l\rangle}\right]A &= \left[\frac{\langle k,\gamma^k,\delta^k\rangle}{\langle k,\alpha^k,\beta^k\rangle};\bot\right]\left(\left[\bot;\frac{\langle l,\gamma^l,\delta^l\rangle}{\langle l,\alpha^l,\beta^l\rangle}\right]A\right)\\ &= \left[\bot;\frac{\langle l,\gamma^l,\delta^l\rangle}{\langle l,\alpha^l,\beta^l\rangle}\right]\left(\left[\frac{\langle k,\gamma^k,\delta^k\rangle}{\langle k,\alpha^k,\beta^k\rangle};\bot\right]A\right).\end{aligned}$$

A local substitution of an index together with its degrees of the EIFIM is defined for the couples of indices (p, k) and/or (q, l), respectively, by

$$\left[\frac{\langle p, \alpha^p, \beta^p\rangle}{\langle k, \alpha^k, \beta^k\rangle}; \perp\right] A$$

$$= \left[(K^* - \{\langle k, \alpha^k, \beta^k\rangle\}) \cup \{\langle p, \alpha^p, \beta^p\rangle\}, L, \{\langle a_{k,l}, \mu_{k_i,l_j}, \nu_{k_i,l_j}\rangle\}\right],$$

$$\left[\perp; \frac{\langle q, \alpha^q, \beta^q\rangle}{\langle l, \alpha^l, \beta^l\rangle}\right] A$$

$$= \left[K^*, (L^* - \{\langle l, \alpha^l, \beta^l\rangle\}) \cup \{\langle q, \alpha^q, \beta^q\rangle\}, L, \{\langle a_{l,l}, \mu_{l_i,l_j}, \nu_{l_i,l_j}\rangle\}\right],$$

$$\left[\frac{\langle p, \alpha^p, \beta^p\rangle}{\langle k, \alpha^k, \beta^k\rangle}; \frac{\langle q, \alpha^q, \beta^q\rangle}{\langle l, \alpha^l, \beta^l\rangle}\right] A = \left[\frac{\langle p, \alpha^p, \beta^p\rangle}{\langle k, \alpha^k, \beta^k\rangle}; \perp\right] \left(\left[\perp; \frac{\langle q, \alpha^q, \beta^q\rangle}{\langle l, \alpha^l, \beta^l\rangle}\right] A\right)$$

$$= \left[\perp; \frac{\langle q, \alpha^q, \beta^q\rangle}{\langle l, \alpha^l, \beta^l\rangle}\right] \left(\left[\frac{\langle p, \alpha^p, \beta^p\rangle}{\langle k, \alpha^k, \beta^k\rangle}; \perp\right] A\right)$$

Obviously, for the above indices k, l, p, q:

$$\left[\frac{\langle k, \alpha^k, \beta^k \rangle}{\langle p, \alpha^p, \beta^p \rangle}; \bot\right] \left(\left[\frac{\langle p, \alpha^p, \beta^p \rangle}{\langle k, \alpha^k, \beta^k \rangle}; \bot\right] A \right)$$

$$= \left[\bot; \frac{\langle l, \alpha^l, \beta^l \rangle}{\langle q, \alpha^q, \beta^q \rangle}\right] \left(\left[\bot; \frac{\langle q, \alpha^q, \beta^q \rangle}{\langle l, \alpha^l, \beta^l \rangle}\right] A \right),$$

$$\left[\frac{\langle k, \alpha^k, \beta^k \rangle}{\langle p, \alpha^p, \beta^p \rangle}; \frac{\langle l, \alpha^l, \beta^l \rangle}{\langle q, \alpha^q, \beta^q \rangle}\right] \left(\left[\frac{\langle p, \alpha^p, \beta^p \rangle}{\langle k, \alpha^k, \beta^k \rangle}; \frac{\langle q, \alpha^q, \beta^q \rangle}{\langle l, \alpha^l, \beta^l \rangle}\right] A \right) = A.$$

Let the sets of indices $P = \{p_1, p_2, ..., p_u\}$, $Q = \{q_1, q_2, ..., q_v\}$ be given. For them we define sequentially:

$$\left[\frac{P^*}{K^*}; \perp\right] A = \left[\frac{\langle p_1, \alpha^{p_1}, \beta^{p_1}\rangle}{\langle k_1, \alpha^{k_1}, \beta^{k_1}\rangle} \dots \frac{\langle p_u, \alpha^{p_u}, \beta^{p_u}\rangle}{\langle k_u, \alpha^{k_u}, \beta^{k_u}\rangle}; \perp\right] A$$

$$\left[\perp; \frac{Q^*}{L^*}\right] A = \left[\perp; \frac{\langle q_1, \alpha^{q_1}, \beta^{q_1}\rangle}{\langle l_1, \alpha^{l_1}, \beta^{l_1}\rangle} \dots \frac{\langle q_v, \alpha^{q_v}, \beta^{q}_{v} q_v\rangle}{\langle l_v, \alpha^{l_v}, \beta^{l_v}\rangle}\right] A,$$

$$\left[\frac{P^*}{K^*}; \frac{Q^*}{L^*}\right] A = \left[\frac{P^*}{K^*}; \perp\right] (\left[\perp; \frac{Q^*}{L^*}\right] A),$$

Obviously, for the sets K, L, P, Q:

$$\left[\frac{K^*}{P^*};\perp\right]\left(\left[\frac{P^*}{K^*};\perp\right]A\right)=\left[\perp;\frac{L^*}{Q^*}\right]\left(\left[\perp;\frac{Q^*}{L^*}\right]A\right)=\left[\frac{K^*}{P^*};\frac{L^*}{Q^*}\right]\left(\left[\frac{P^*}{K^*};\frac{Q^*}{L^*}\right]\right.$$

From the above definitions, it follows the validity of the following **Theorem.** For every four sets of indices $P_1^*, P_2^*, Q_1^*, Q_2^*$

$$\left[\frac{P_2^*}{P_1^*};\frac{Q_2^*}{Q_1^*}\right]\left[\frac{P_1^*}{K^*};\frac{Q_1^*}{L^*}\right]A=\left[\frac{P_2^*}{K^*};\frac{Q_2^*}{L^*}\right]A.$$

Finally, for $k \in K$ and $l \in L$ and element $\langle \mu_{k,l}, \nu_{k,l} \rangle$ in the EIFIM A, we define the substitution

$$\left\{\frac{\langle k, l \rangle}{\langle \zeta, \eta \rangle}\right\} A$$

and in a result we obtain the EIFIM that has element $\langle \zeta, \eta \rangle$ in its k-th row and l-th column instead of element $\langle \mu_{k,l}, \nu_{k,l} \rangle$.

Obviously,

$$\left\{\frac{\langle k,l\rangle}{\langle \mu_{k,l},\nu_{k,l}\rangle}\right\}\left(\left\{\frac{\langle k,l\rangle}{\langle \zeta,\eta\rangle}\right\}A\right)=A.$$

Conclusion

Here, new operations, relations and operators over two EIFIMs are introduced. These operations are specific for this type of IMs and they cannot be transformed over the standard IMs or over IFIMs because of the lack of IFPs associated to the indices of these IMs. The shown EIFIM representations are suitable for example, for oriented, as well as for non-oriented graphs. The new operations, relations and operators can be implemented in different types of data bases, including the big data.

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