Intuitionistic fuzzy coloured multigraphs and their index matrix representation

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Introduction

The concept of an Intuitionistic Fuzzy Graph (IFG) was introduced in September 1994. Only a month later - in October 1994 the IFGs obtained Index Matrix (IM) representation. Ten years later, the concept of an intuitionistic fuzzy multigraph was introduced and its IM-interpretation was discussed. These graphs are based on the concepts of an Intuitionistic Fuzzy Sets (IFSs) and Intuitionistic Fuzzy Pairs (IFPs).

In 1987, the concept of an Index Matrix (IM) was introduced as an extension of the well-known object "matrix". the IMs (that were defined initially as two-dimensional objects) were extended to three- and n-dimensional IMs, as well as to Intuitionistic Fuzzy Index Matrices (IFIMs) and to Extended IFIMs (EIFIMs).
In the present paper we discuss the concept of an Intuitionistic Fuzzy Coloured MultiGraph (IFCMG), show its EIFIM-representation and some of its properties.

Short remarks on EIFIMs

The ordered pair $\langle a, b\rangle$ is called an IFP when $a, b, a+b \in [0,1]$. When there is a fixed universe E and its subset A, the corresponding IFS is an object with the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

is called an IFS.

Let I be a fixed set. By EIFIM with index sets K and L $(K, L \subset I)$, we denote the object:

$$[K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}] \equiv$$

	$l_1, \langle \alpha_1^L, \beta_1^L \rangle$	$\ldots$	$l_j, \langle \alpha_j^L, \beta_j^L \rangle$	$\ldots$	$l_n, \langle \alpha_n^L, \beta_n^L \rangle$
$k_1, \langle \alpha_1^K, \beta_1^K \rangle$	$\langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle$	$\ldots$	$\langle \mu_{k_1,l_j}, \nu_{k_1,l_j} \rangle$	$\ldots$	$\langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$k_i, \langle \alpha_i^K, \beta_i^K \rangle$	$\langle \mu_{k_i,l_1}, \nu_{k_i,l_1} \rangle$	$\ldots$	$\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle$	$\ldots$	$\langle \mu_{k_i,l_n}, \nu_{k_i,l_n} \rangle$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$k_m, \langle \alpha_m^K, \beta_m^K \rangle$	$\langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle$	$\ldots$	$\langle \mu_{k_m,l_j}, \nu_{k_m,l_j} \rangle$	$\ldots$	$\langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle$

,

where $K = \{k_1, k_2, ..., k_m\}$, $L = \{l_1, l_2, ..., l_n\}$ and

$$\begin{aligned} K^* &= \{\langle k_i, \alpha_i^K, \beta_i^K \rangle | k_i \in K\} = \{\langle k_i, \alpha_i^K, \beta_i^K \rangle | 1 \leq i \leq m\}, \\ L^* &= \{\langle l_j, \alpha_j^L, \beta_j^L \rangle | l_j \in L\} = \{\langle l_j, \alpha_j^L, \beta_j^L \rangle | 1 \leq j \leq n\} \end{aligned}$$

are IFSs and for every $1 \leq i \leq m, 1 \leq j \leq n$:

$$\begin{aligned} \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} &\in [0,1], \\ \alpha_i^K, \beta_i^K, \alpha_i^K + \beta_i^K &\in [0,1], \\ \alpha_j^L, \beta_j^L, \alpha_j^L + \beta_j^L &\in [0,1]. \end{aligned}$$

We will describe two operations over EIFIMs that are extensions of the standard operations over IMs, IFIMs and EIFIMs.
Let everywhere below $\circ, * \in \{\vee, \wedge\}$ and when $\circ$ or $*$ is $\vee$, then $\circ_1$ or $*_1$ is "$\max$", and $\circ_2$ or $*_2$ is "$\min$"; and when $\circ$ or $*$ is $\wedge$, then $\circ_1$ or $*_1$ is "$\min$", and $\circ_2$ or $*_2$ is "$\max$". Let $e_\vee = \langle 0, 1\rangle$ and $e_\wedge = \langle 1, 0\rangle$.
For the EIFIMs A and B operations that are analogous to the standard matrix operations of addition and multiplication are defined as follows:

Addition $\oplus^*_\circ$

$$A \oplus^*_\circ B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$\begin{aligned} T^* &= K^* \cup P^* = \{\langle t_u, \alpha^t_u, \beta^t_u \rangle | t_u \in K \cup P\}, \\ V^* &= L^* \cup Q^* = \{\langle v_w, \alpha^v_w, \beta^v_w \rangle | v_w \in L \cup Q\}, \end{aligned}$$

$$\alpha^t_u = \begin{cases} \alpha^k_i, & \text{if } t_u \in K - P \\ \alpha^p_r, & \text{if } t_u \in P - K \\ *_1(\alpha^k_i, \alpha^p_r), & \text{if } t_u \in K \cap P \end{cases},$$

$$\beta^v_w = \begin{cases} \beta^l_j, & \text{if } v_w \in L - Q \\ \beta^q_s, & \text{if } t_w \in Q - L \\ *_2(\beta^l_j, \beta^q_s), & \text{if } t_w \in L \cap Q \end{cases},$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$\begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ \langle \sigma_{p_r,q_s}, \tau_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ \langle \circ_1(\mu_{k_i,l_j}, \sigma_{p_r,q_s}), \circ_2(\nu_{k_i,l_j}, \tau_{p_r,q_s}) \rangle, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q; \\ e_\circ, & \text{otherwise.} \end{cases}$$

Termwise multiplication $\otimes^*_\circ$

$$A \otimes^*_\circ B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$\begin{aligned}
T^* &= K^* \cap P^* = \{\langle t_u, \alpha^t_u, \beta^t_u \rangle | t_u \in K \cap P\}, \\
V^* &= L^* \cap Q^* = \{\langle v_w, \alpha^v_w, \beta^v_w \rangle | v_w \in L \cap Q\}, \\
\alpha^t_u &= *_1(\alpha^k_i, \alpha^p_r), \text{ for } t_u = k_i = p_r \in K \cap P, \\
\beta^v_w &= *_2(\beta^l_j, \beta^q_s), \text{ for } v_w = l_j = q_s \in L \cap Q
\end{aligned}$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \circ_1(\mu_{k_i,l_j}, \sigma_{p_r,q_s}), \circ_2(\nu_{k_i,l_j}, \tau_{p_r,q_s}) \rangle.$$

Short remark on coloured multigraphs and intuitionistic fuzzy graphs

Let $\mathcal{V}$ be a finite set of vertex identifiers. Let $\mathcal{L}_\mathcal{V}$ and $\mathcal{L}_\mathcal{H}$ be a bounded lattices of, respectively, vertex and edge colours, with operations $\cap$, and $\cup$, least element $\bot_\mathcal{L}$ and $\bot_\mathcal{H}$, and greatest element $\top_\mathcal{L}$ and $\top_\mathcal{H}$, respectively. These operations describe the prescribed combination of vertex and edge labels and their interpretation can be algebraic. Let $\bot \notin \mathcal{L}_\mathcal{V} \cup \mathcal{L}_\mathcal{H}$ denote the absence of an edge in the corresponding colour or, as we used it above.

The Colour MultiGraph (CMG) will be called the object $G = \langle V^*, H^* \rangle$ for which:

- $V^* \subseteq \mathcal{V} \times \mathcal{L}_{\mathcal{V}}$,
- $\alpha = |V|$ - the cardinality of the set of the G-vertices $V \subseteq \mathcal{V}$ and $\alpha \leq |\mathcal{V}|$,
- $V^* = \{\langle v_i, c_1 \rangle, \cdots, \langle v_\alpha, c_\alpha \rangle\}$,
- $\mathcal{H}^* \subseteq \mathcal{V} \times \mathcal{V} \times \mathcal{N} \times \{0,1\}^n$,
- $\mathcal{N}$ is the set of the natural numbers,
- where $h_{i,j}$ is the number of the edges between vertices v_i and v_j of V,
- n is the number of all colours used for colouring of the edges of G and $n \leq |\mathcal{L}_{\mathcal{H}}|$,
- $\mathcal{H}^* = \{\langle v_i, v_j, h_{i,j}, \lambda(c^1_{i,j}), \cdots, \lambda(c^n_{i,j}) \rangle | v_i, v_j \in V \ \& \ (\forall k : 1 \leq k \leq n)(c^k_{i,j} \in \mathcal{L}_{\mathcal{H}})\}$.

Therefore, we can interpret the set $\mathcal{L}_{\mathcal{H}}$ as a fixed palette of edge colours for the graphs with vertices from $\mathcal{V}$ and for each k $(1 \leq k \leq n)$, for every two vertices v_i, v_j $(1 \leq i, j \leq \alpha)$:

$$\lambda(c) = \begin{cases} 1, & \text{if } c \in \mathcal{L}_{\mathcal{H}} \text{ is a colour of an edge between } v_i \text{ and } v_j \\ 0, & \text{if there is no edge} \end{cases}.$$

Then

$$h_{i,j} = \sum_{k=1}^{n} \lambda(c_{i,j}^k).$$

The IFG is a graph which vertices and edges are indexed by IFPs. Having in mind the relationship between the colours and IFSs, we can see immediately that each CMG (and, of course, each standard colour graph) can be interpreted as an IFG. Really, all colours without black, white and their nuances can be represent by IFPs (see Fig. 1)

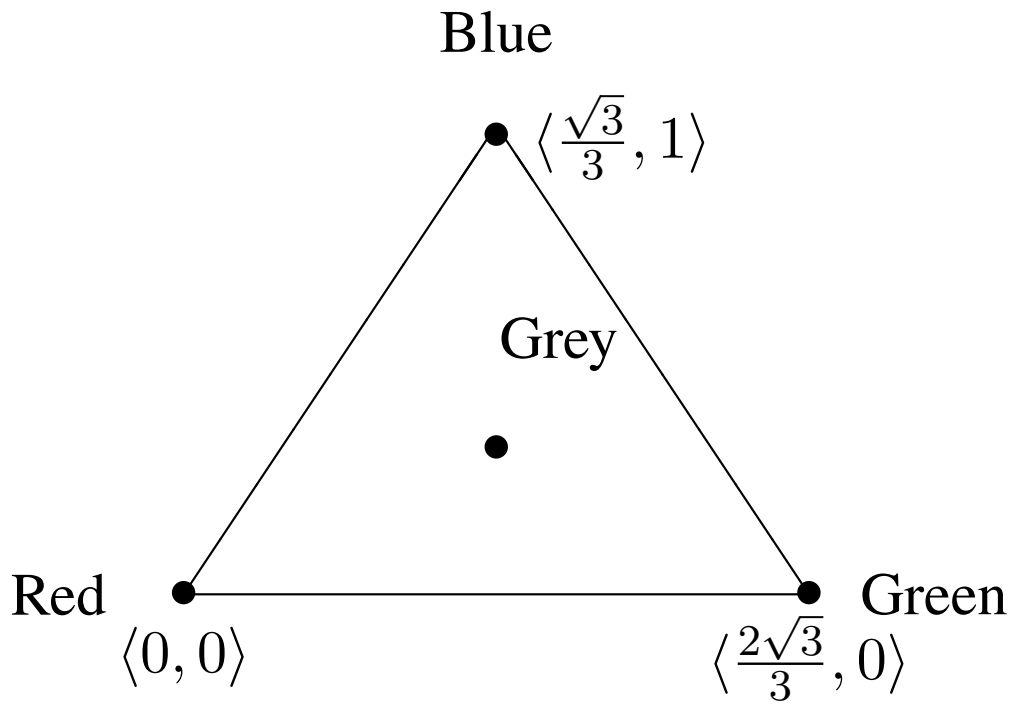

Blue
$\langle \frac{\sqrt{3}}{3}, 1 \rangle$
Grey
Red
Green
$\langle 0, 0 \rangle$
$\langle \frac{2\sqrt{3}}{3}, 0 \rangle$

Fig. 1.

When we like to represent black and white colours, and all their nuances we can use Temporal IFS where temporal scale corresponds to the section with vertices “black” and “white” on Fig. 2. In this figure all colours without black, white and their nuances and with only gray colour lie in the basis plane determined by points “red”, “green” and “blue”, and with point “gray”.

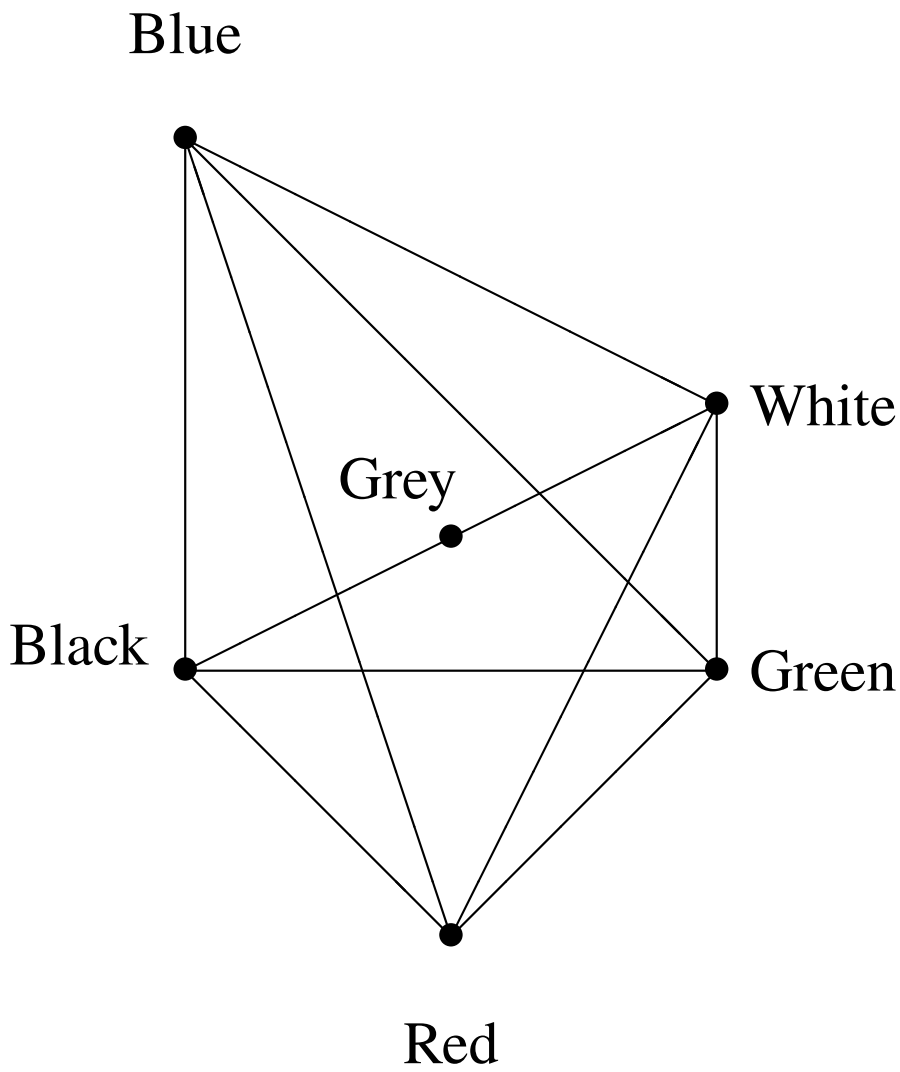


Fig. 2.

The triangles that lie in planes parallel to the basis plane close to vertex “black” will contain all colours from the basis plane to which some quantity of black colour is added. Respectively, the triangles that lie in planes parallel to the basis plane close to vertex “white” will contain all colours from the basis plane to which some quantity of white colour is added.
In the present research, for brevity, we will work with the colours in the basis plane that obviously can be represented by IFPs (see Fig. 3).

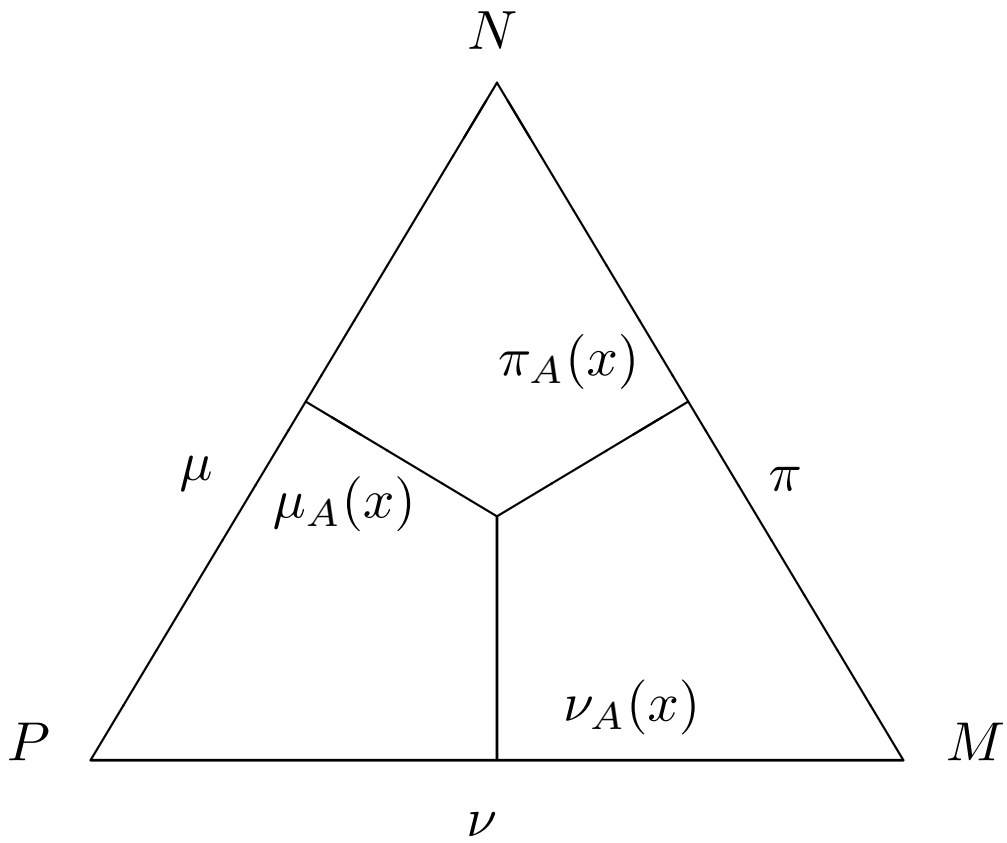


Fig. 3.

intuitionistic fuzzy coloured multigraphs

Now, we can define: Intuitionstoc fuzzy coloured multigraph (IFCMG) is a CMG which vertices and edges are indexed by IFPs. It can be defined as $G = \langle V^*, H^* \rangle$, where set V has the above sense, but

$$V^* = \{\langle v_1, c_1, \mu_{v_1}, \nu_{v_1} \rangle, \cdots, \langle v_\alpha, c_\alpha, \mu_{v_\alpha}, \nu_{v_\alpha} \rangle\},$$

where the IFP $\langle \mu_{v_k}, \nu_{v_k} \rangle$ represents, e.g., the degrees of existence and non-existence of the k-th vertex v_k of the graph (for $1 \leq k \leq \alpha$). For the set of edges H^* there are two cases that determine the type of the IFCMG.

For the Simple IFCMG (SIFCMG), H^* has the above form, but

$$\lambda(c_{i,j}^k) = \begin{cases} \langle \rho_{c_{i,j}^k}, \sigma_{c_{i,j}^k} \rangle, & \text{if } \langle \rho_{c_{i,j}^k}, \sigma_{c_{i,j}^k} \rangle \text{ is the IFP that corresponds} \\ & \quad \text{to the colour } c_{i,j}^k \in \mathcal{L}_{\mathcal{H}} \\ & \quad \text{of an edge between } v_i \text{ and } v_j \\ 0, & \text{if there is not an edge} \end{cases}.$$

For the Complex IFCMG (CIFCMG), H^* has the form

$$\begin{aligned} \mathcal{H}^{\#} &= \{\langle v_i, v_j, h_{i,j}, \lambda(c_{i,j}^1), \langle \mu_{c_{i,j}^1}, \nu_{c_{i,j}^1} \rangle, \cdots, \lambda(c_{i,j}^n), \langle \mu_{c_{i,j}^n}, \nu_{c_{i,j}^n} \rangle \rangle | \\ & \quad v_i, v_j \in V \& \ (\forall k : 1 \leq k \leq n)(c_{i,j}^k \in \mathcal{L}_{\mathcal{H}})\}. \end{aligned}$$

Therefore, in the complex case, the graph has as coloured edges, as well as degrees of existence and of non-existence of these colours. If the k-th edge between vertices v_i and v_j has a colour $c_{i,j}^k$ with a corresponding IFP $\langle \rho_{c_{i,j}^k}, \sigma_{c_{i,j}^k} \rangle$ and if this is valid for each edge of the graph, then we can transform the CIFCMG to a SIFCMG as follows: replace set $H^{\#}$ with H^*, but now function λ has the form

$$\lambda(c_{i,j}^k) = \begin{cases} O_{\rho_{c_{i,j}^k}, \sigma_{c_{i,j}^k}}(\langle \mu_{c_{i,j}^1}, \nu_{c_{i,j}^1} \rangle), & \text{if } \langle \rho_{c_{i,j}^k}, \sigma_{c_{i,j}^k} \rangle \text{ is the IFP that} \\ & \text{corresponds to the colour} \\ & c_{i,j}^k \in \mathcal{L}_{\mathcal{H}} \text{ of an edge between} \cdot \\ & v_i \text{ and } v_j \\ 0, & \text{if there is not an edge,} \end{cases}$$

where the intuitionistic fuzzy operator $O_{\alpha,\beta}$ ($\alpha, \beta \in [0,1]$) is defined over the IFP $\langle a, b \rangle$ and it has one of the following forms:

$$
\begin{aligned}
F_{\alpha,\beta}(\langle a,b\rangle) &= \langle a+\alpha(1-a-b), b+\beta(1-a-b)\rangle \text{ and } \alpha+\beta\leq 1\\
G_{\alpha,\beta}(\langle a,b\rangle) &= \langle \alpha a, \beta b\rangle\\
H_{\alpha,\beta}(\langle a,b\rangle) &= \langle \alpha a, b+\beta(1-a-b)\rangle\\
H^*_{\alpha,\beta}(\langle a,b\rangle) &= \langle \alpha a, b+\beta(1-\alpha a-b)\rangle\\
J_{\alpha,\beta}(\langle a,b\rangle) &= \langle a+\alpha(1-a-b), \beta b\rangle\\
J^*_{\alpha,\beta}(\langle a,b\rangle) &= \langle a+\alpha(1-a-\beta b), \beta b\rangle
\end{aligned}
$$

for every $a, b, \alpha, \beta \in [0,1]$ such that $a+b\leq 1, \alpha, \beta \leq 1$.
As a result of using such an operator, we can change the color of a given rainbow. For example, the bright green colour of an edge may become petrol green.

Representation of SIFCMGs and CIFCMGs by EIFIMs

When we have a SIFCMG we can bijective juxtapose to it the following EIFIM:

$$C = [V^*, V^*, \overline{N_1}, \langle h_{i,j}, \lambda(c^1_{i,j}), \cdots, \lambda(c^n_{i,j})\}\rangle],$$

where

$$\overline{N_1} = \{c^1, \cdots, c^n, \perp\}$$

are the sets of all colours with which the edges of G_1 and G_2 are coloured, respectively, and the remaining components are described above. Then C has the form

$$
\left\{
\begin{array}{c|ccccc}
\perp & v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & \dots & v_j, \langle \mu_{v_j}, \nu_{v_j} \rangle & \dots & v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle \\
\hline
v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & h_{1,1} & \dots & h_{1,j} & \dots & h_{1,\alpha} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_i, \langle \mu_{v_i}, \nu_{v_i} \rangle & h_{i,1} & \dots & h_{i,j} & \dots & h_{i,\alpha} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle & h_{\alpha,1} & \dots & h_{\alpha,j} & \dots & h_{\alpha,\alpha}
\end{array}
\right\},
$$

$$
\begin{array}{c|cccccc}
c^1 & v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & \dots & v_j, \langle \mu_{v_j}, \nu_{v_j} \rangle & \dots & v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle \\
\hline
v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & \langle \rho_{c^1_{1,1}}, \sigma_{c^1_{1,1}} \rangle & \dots & \langle \rho_{c^1_{1,j}}, \sigma_{c^1_{1,j}} \rangle & \dots & \langle \rho_{c^1_{1,\alpha}}, \sigma_{c^1_{1,\alpha}} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_i, \langle \mu_{v_i}, \nu_{v_i} \rangle & \langle \rho_{c^1_{i,1}}, \sigma_{c^1_{i,1}} \rangle & \dots & \langle \rho_{c^1_{i,j}}, \sigma_{c^1_{i,j}} \rangle & \dots & \langle \rho_{c^1_{i,\alpha}}, \sigma_{c^1_{i,\alpha}} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle & \langle \rho_{c^1_{\alpha,1}}, \sigma_{c^1_{\alpha,1}} \rangle & \dots & \langle \rho_{c^1_{\alpha,j}}, \sigma_{c^1_{\alpha,j}} \rangle & \dots & \langle \rho_{c^1_{\alpha,\alpha}}, \sigma_{c^1_{\alpha,\alpha}} \rangle
\end{array} ,
$$

$$
\dots ,
$$

$$
\left.\begin{array}{c|cccccc}
c^n & v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & \cdots & v_j, \langle \mu_{v_j}, \nu_{v_j} \rangle & \cdots & v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle \\
\hline
v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & \langle \rho_{c^n_{1,1}}, \sigma_{c^n_{1,1}} \rangle & \cdots & \langle \rho_{c^n_{1,j}}, \sigma_{c^n_{1,j}} \rangle & \cdots & \langle \rho_{c^n_{1,\alpha}}, \sigma_{c^n_{1,\alpha}} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_i, \langle \mu_{v_i}, \nu_{v_i} \rangle & \langle \rho_{c^n_{i,1}}, \sigma_{c^n_{i,1}} \rangle & \cdots & \langle \rho_{c^n_{i,j}}, \sigma_{c^n_{i,j}} \rangle & \cdots & \langle \rho_{c^n_{i,\alpha}}, \sigma_{c^n_{i,\alpha}} \rangle \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle & \langle \rho_{c^n_{\alpha,1}}, \sigma_{c^n_{\alpha,1}} \rangle & \cdots & \langle \rho_{c^n_{\alpha,j}}, \sigma_{c^n_{\alpha,j}} \rangle & \cdots & \langle \rho_{c^n_{\alpha,\alpha}}, \sigma_{c^n_{\alpha,\alpha}} \rangle
\end{array}\right\}.
$$

When we have a SIFCMG we can bijective juxtapose to it the following EIFIM:

$$C = [V^*, V^*, \overline{N_1}, \langle h_{i,j}, (\lambda(c^1_{i,j}), \langle \mu_{c^1_{i,j}}, \nu_{c^n_{i,j}} \rangle), \cdots, (\lambda(c^n_{i,j}), \langle \mu_{c^n_{i,j}}, \nu_{c^n_{i,j}} \rangle \} \rangle],$$

then C has the form

$$
\left\{
\begin{array}{c|ccccc}
\perp & v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & \dots & v_j, \langle \mu_{v_j}, \nu_{v_j} \rangle & \dots & v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle \\
\hline
v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle & h_{1,1} & \dots & h_{1,j} & \dots & h_{1,\alpha} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_i, \langle \mu_{v_i}, \nu_{v_i} \rangle & h_{i,1} & \dots & h_{i,j} & \dots & h_{i,\alpha} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle & h_{\alpha,1} & \dots & h_{\alpha,j} & \dots & h_{\alpha,\alpha}
\end{array}
\right\},
$$

c^1	$v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle$	$\dots$	$v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle$
$v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle$	$(\langle \rho_{c^1_{1,1}}, \sigma_{c^1_{1,1}} \rangle, \langle \mu_{c^1_{1,1}}, \nu_{c^n_{1,1}} \rangle)$	$\dots$	$(\langle \rho_{c^1_{1,\alpha}}, \sigma_{c^1_{1,\alpha}} \rangle, \langle \mu_{c^1_{1,\alpha}}, \nu_{c^1_{1,\alpha}} \rangle)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$v_i, \langle \mu_{v_i}, \nu_{v_i} \rangle$	$(\langle \rho_{c^1_{i,1}}, \sigma_{c^1_{i,1}} \rangle, \langle \mu_{c^1_{i,1}}, \nu_{c^1_{i,1}} \rangle)$	$\dots$	$(\langle \rho_{c^1_{i,\alpha}}, \sigma_{c^1_{i,\alpha}} \rangle, \langle \mu_{c^1_{i,\alpha}}, \nu_{c^1_{i,\alpha}} \rangle)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle$	$(\langle \rho_{c^1_{\alpha,1}}, \sigma_{c^1_{\alpha,1}} \rangle, \langle \mu_{c^1_{\alpha,1}}, \nu_{c^1_{\alpha,1}} \rangle)$	$\dots$	$(\langle \rho_{c^1_{\alpha,\alpha}}, \sigma_{c^1_{\alpha,\alpha}} \rangle \langle \mu_{c^1_{i,\alpha}}, \nu_{c^1_{i,\alpha}} \rangle)$

$$\dots,$$

c^n	$v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle$	$\dots$	$v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle$
$v_1, \langle \mu_{v_1}, \nu_{v_1} \rangle$	$(\langle \rho_{c^n_{1,1}}, \sigma_{c^n_{1,1}} \rangle, \langle \mu_{c^n_{1,1}}, \nu_{c^n_{1,1}} \rangle)$	$\dots$	$(\langle \rho_{c^n_{1,\alpha}}, \sigma_{c^n_{1,\alpha}} \rangle, \langle \mu_{c^n_{1,\alpha}}, \nu_{c^n_{1,\alpha}} \rangle)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$v_i, \langle \mu_{v_i}, \nu_{v_i} \rangle$	$(\langle \rho_{c^n_{i,1}}, \sigma_{c^n_{i,1}} \rangle, \langle \mu_{c^n_{i,1}}, \nu_{c^n_{i,1}} \rangle)$	$\dots$	$(\langle \rho_{c^n_{i,\alpha}}, \sigma_{c^n_{i,\alpha}} \rangle, \langle \mu_{c^n_{i,\alpha}}, \nu_{c^n_{i,\alpha}} \rangle)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$v_\alpha, \langle \mu_{v_\alpha}, \nu_{v_\alpha} \rangle$	$(\langle \rho_{c^n_{\alpha,1}}, \sigma_{c^n_{\alpha,1}} \rangle, \langle \mu_{c^n_{\alpha,1}}, \nu_{c^n_{\alpha,1}} \rangle)$	$\dots$	$(\langle \rho_{c^n_{\alpha,\alpha}}, \sigma_{c^n_{\alpha,\alpha}} \rangle \langle \mu_{c^n_{i,\alpha}}, \nu_{c^n_{i,\alpha}} \rangle)$

Now, using the operations addition and termwise multiplication over EIFIMs, described in Section 2, we can bijectively realize the operations union and intersection of two CMGs.
We can define object $\mathcal{G}(\mathcal{V}, \mathcal{L}_{\mathcal{V}}, \mathcal{L}_{\mathcal{H}})$ as the class of CMGs defined in Section 3, with vertex sets contained in $\mathcal{V}$. Let $G_{\emptyset}$ be the graph with no vertices and no edges. Let G_{full} have vetrex set $\mathcal{U}$, vertex colour at every vertex, and one edge of each colour in C_H for every pair of distinct vertices.
Therefore, we can prove by the following theorem.
Theorem. The structures $\langle \mathcal{G}(\mathcal{V}, \mathcal{L}_{\mathcal{V}}, \mathcal{L}_{\mathcal{H}}), \oplus, G_{\emptyset} \rangle$ and $\langle \mathcal{G}(\mathcal{V}, \mathcal{L}_{\mathcal{V}}, \mathcal{L}_{\mathcal{H}}), \otimes, G_{\text{full}} \rangle$ are commutative monoids. Consequently, the corresponding EIFIM-representations form commutative monoids under operations $\oplus$ and $\otimes$, respectively.

Conclusion

The present paper is the first step to realize the idea to introduce IFCMG.
In the next study, we will discuss other operations, as well as relations and modal and level type operators over CMGs and their IM-representations.
The basic concepts related to graphs (e.g. path, diameter, etc.) and properties (e.g. planarity, (dis)connectedness, symmetry, etc.) of IFCMGs in their IM-forms will be discussed.

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