BIOAUTOMATION, 2009, **13** (2), 61-72

Preferences based Control Design of Complex Fed-batch Cultivation Process

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Received: July 2, 2009 **Accepted: July 7, 2009**

Published: August 5, 2009

Abstract: *In the paper is presented preferences based control design and stabilization of the growth rate of fed-batch cultivatiion processes. The control is based on an enlarged Wang-Monod-Yerusalimsky kinetic model. Expected utility theory is one of the approaches for utilization of conceptual information (expert preferences). In the article is discussed utilization of stochastic machine learning procedures for evaluation of expert utilities as criteria for optimization.*

Keywords: *Preferences, Utility function, Fed-batch, Yerusalimsky kinetic, Brunovsky normal form, Optimal control.*

Introduction

The incorporation of human preferences in complex systems is a contemporary trend in the scientific investigations. The aim is to develop iterative control design with a merger of empirical knowledge (subjective preferences) with the mathematical exactness and optimal control algorithms [1, 8]. People preferences contain characteristic of uncertainty due to the cardinal type of the empirical expert information. The appearance of this uncertainty has subjective and probabilistic nature. Probability theory and expected Utility theory address decision making under these conditions in mathematics [2, 5]. The necessity of a merger of empirical knowledge with mathematical exactness causes difficulties. A possible approach for solving these problems is the stochastic approximation. The uncertainty of the subjective preferences may be viewed as a noise that could be eliminated as typical for the stochastic approximation procedures and machine learning [8]. A main requirement of the stochastic assessment is the representation of the qualitative nature of preferences.

This article deals with an investigation of Monod and Yerusalimsky kinetic models with an approach based both on the differential geometry and on the optimal control theory [1, 6]. This approach permits new control solutions for some optimal control problems in the field. Stochastic algorithms for assessment of Decision-maker's (DM) expected utility function on the basis of the expressed expert preferences are included in the control design.

The objective of this paper is to present comfortable tools and mathematical methodology that are useful for dealing with the uncertainty of human behavior in complex control problems. The incomplete information is some times compensated with participation of imprecise human estimations. This article deals with a possible mathematical description of the system "technologist-fed-batch process". The dialogue "DM's preferences – computer" realizes a machine learning.

Description of Wang-Yerusalimsky kinetic model

Unstructured biotechnological models take cell mass as a uniform quality without internal dynamic. The reaction rates depend only upon the macroscopic conditions in the liquid phase of the bioreactor. Mathematical unstructured models of fed-batch process can be written based on mass balance equation [7, 12]. Below we investigate an enlarged form of the Yerusalimsky kinetic model (Wang-Yerusalimsky model [10, 12]):

$$\begin{aligned}
\dot{X} &= \mu X - \frac{F}{V} X, \\
\dot{S} &= -k\mu X + (S_0 - S)\frac{F}{V}, \\
\dot{\mu} &= m\left(\mu_m \frac{S}{(K_S + S)} \frac{k_E}{(k_E + X)} - \mu \right), \\
\dot{V} &= F, \\
\dot{E} &== k_2 \mu E - \frac{F}{V} E, \\
\dot{A} &== k_3 \mu X - \frac{F}{V} A,
\end{aligned} \tag{1}$$

where X is the concentration of biomass, g·l^{-1}; S – the concentration of substrate (glucose), g·l^{-1}; V – bioreactor volume, l; F – substrate feed rate, h^{-1}; S_0 – substrate concentration in the feed, g·l^{-1}; $\mu_{ma\,x}$ – maximum specific growth rate, h^{-1}; K_S – saturation constant, g·l^{-1}; k, k_2 , k_3 and k_E – constant, g·g^{-1}; m – coefficient, - ; E – the concentration of ethanol, g·l^{-1}; A – the concentration of acetate, g·l^{-1}.

We preserve the notation $U(.)$ for the criteria for optimization (*a unimodal polynomial expert utility function*). The parameters are as follows: μ_m = 0.59 h^{-1}; K_S = 0.045 g·l^{-1}; m = 3; S_0 = 100 g·l^{-1}; k = 2; k_2 = 3.79; k_3 = 1/71; k_E = 50; F_{max} = 0.19 h^{-1}; V_{max} = 1.5 l. The dynamics of μ is modeled as a first order lag process with rate constant m, in response to the deviation in μ. The 5th equation describes the production of ethanol (E). The last equation describes the production of acetate (A). The first and the last equations become dynamically equivalent with a simple transformation:

$$X = \frac{1}{k_3} A \tag{2}$$

That is why in the non-linear kinetic model we omit the last equation. The initial values of the state variables are: $X_i(0)$ = 0.99; $S_i(0)$ = 0.01; $\mu_i(0)$ = 0.1; $E_I(0)$ = 0.1; $V_i(0)$ = 0.5. The parameters are taken from different sources [9, 10].

Expected utility, formulations and evaluation

Standard description of the utility function application is presented by Fig 1. There are a variety of final results that are consequence of the expert or DM activity and choice. This activity is motivated by a technological objective that possibly includes economical, social, ecological or other important characteristics. A utility function $u(.)$ assesses each of this final results (x_i, i = 1÷n). The DM judgment of the process behavior based on the DM choice is measured quantitatively by the following equation:

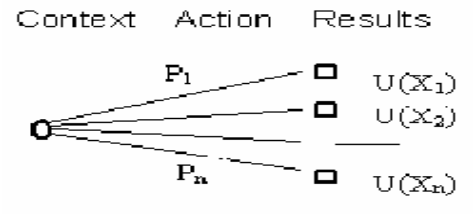

Fig. 1 Utility application

$$u(p) = \sum_i p_i u(x_i), \text{ where } p = (p_1, p_2, .., p_i, .. p_n), \sum_i p_i = 1 \quad (3)$$

We denote with p_i subjective or objective probabilities that reflect the uncertainty of the final result.

The strong mathematical formulation is the following. Let ***Z*** is a set of alternatives and ***P*** is a subset of discrete probability distributions over ***Z***. A utility function is any function $u(.)$ which fulfils:

$$(p \succ q, (p, q) \in \boldsymbol{P}^2) \Leftrightarrow (\int u(.)dp > \int u(.)dq), (p, q) \in \boldsymbol{P}^2 \quad (4)$$

The DM's preference relation over ***P***, $\boldsymbol{Z} \subseteq \boldsymbol{P}$ is expressed by ($\succ$). Its induced indifference relation ($\sim$) is defined thus: $((x \sim y) \Leftrightarrow \neg((x \succ y) \vee (x \prec y)), (x, y) \in \boldsymbol{Z}^2)$. The existence of an utility function $u(.)$ over ***Z*** determines the preference relation ($\succ$) as a negatively transitive and asymmetric one [2, 5]. We mark the lottery "appearance of the alternative (x) with probability α and appearance of the alternative (y) with probability ($1-\alpha$)" as $<x, y, \alpha>$. It is assumed that an utility function $u(.)$ exists and that is fulfilled $((q,p) \in \boldsymbol{P}^2 \Rightarrow (\alpha q + (1-\alpha)p) \in \boldsymbol{P}$, for $\forall \alpha \in [0,1])$. These conditions determine the utility function with precision up to an affine scale (interval scale), $\mathbf{u_1(.) \sim u_2(.) \Leftrightarrow u_1(.) = a u_2(.) + b}$, $\boldsymbol{a > 0}$ [2, 8]. The following notations will be used $\mathbf{A_u = \{(\alpha, x, y, z) / (\alpha u(x) + (1-\alpha) u(y)) > u(z)\}}$ and $\mathbf{(B_u = \{(\alpha, x, y, z) / (\alpha u(x) + (1-\alpha) u(y)) < u(z)\})}$. The expected DM utility is constructed by pattern-recognition of $\mathbf{A_u}$ and $\mathbf{B_u}$ [8]. Key element is the next proposition [8]:

PROPOSITION 1: If $\mathbf{A_{u1} = A_{u2}}$ than $\mathbf{u_1(.) = a u_2(.) + b}$, $\boldsymbol{a > 0}$.

The following presents the procedure for evaluation of the utility functions:
The DM compares the "lottery" $<x, y, \alpha>$ with the simple alternative z, $z \in \boldsymbol{Z}$ ***("better-" $\succ$, $f(x, y, z, \alpha) = 1$", "worse-" $\prec$, $f(x, y, z, \alpha) = -1$" or "can't answer or equivalent-" $\sim$, $f(x, y, z, \alpha) = 0$", $f(.)$*** *denotes the qualitative DM answer)*. *This determins a learning point* $\mathbf{((x, y, z, \alpha), f(x, y, z, \alpha))}$. *The next recurrent stochastic algorithm constructs the utility polynomial approximation* $u(x) = \sum_i c_i \Phi_i(x)$:

$$c_i^{n+1} = c_i^n + \gamma_n \left[f(t^{n+1}) - \overline{(c^n, \Psi(t^{n+1}))} \right] \Psi_i(t^{n+1}), \quad (5)$$

$$\sum_n \gamma_n = +\infty, \sum_n \gamma_n^2 < +\infty, \forall n, \gamma_n > 0 .$$

In the equation are used the following notations (based on $\mathbf{A_u}$): $\mathbf{t} = (x, y, z, \alpha)$, $\psi_i(t) = \psi_i(x, y, z, \alpha) = \alpha\Phi_i(x) + (1-\alpha)\Phi_i(y) - \Phi_i(z)$, where $\Phi_i(x)$ is a family of polynomials. The

line above $\bar{y} = \overline{(c^n, \Psi(t))}$ means $\bar{y} = 1$ if y > 1, $\bar{y} = (-1)$ if y < (–1) and $\bar{y}$=**y** if (–1) < y< 1 [8]. The learning points are set with a pseudo random sequence [11].

The expert relates intuitively the "learning point" (x, y, z, α)) to the set $\mathbf{A_u}$ with probability $D_1(x, y, z, \alpha)$ or to the set $\mathbf{B_u}$ with probability $D_2(x, y, z, \alpha)$. The probabilities $D_1(x, y, z, \alpha)$ and $D_2(x, y, z, \alpha)$ are mathematical expectation of f(.) over $\mathbf{A_u}$ and $\mathbf{B_u}$, respectively, $D_1(x, y, z, \alpha) = M(f/x, y, z, \alpha)$, if $M(f/x, y, z, \alpha) > 0$, $D_2(x, y, z, \alpha) = (-M(f/x, y, z, \alpha))$, if $M(f/x, y, z, \alpha) < 0$. Let $D'(x, y, z, \alpha)$ be the random value: $D'(x, y, z, \alpha) = D_1(x, y, z, \alpha)$, $M(f/x, y, z, \alpha) > 0$; $D'(x, y, z, \alpha) = (-D_2(x, y, z, \alpha))$, $M(f/x, y, z, \alpha) < 0$; $D'(x, y, z, \alpha) = 0$, $M(f/x, y, z, \alpha) = 0$. We approximate $D'(x, y, z, \alpha)$ by a function of the type $G(x, y, z, \alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$, where $g(x)=\sum_i c_i \Phi_i(x)$. **The function g(x) is an approximation of the utility *u*(.).** The coefficients c_i^n take part in the decomposition $g^n(x) = \sum_{i=1}^{N} c_i^n \Phi_i(x)$ and $(c^n, \Psi(t)) = \alpha g^n(x) + (1-\alpha) g^n(y) - g^n(z) = G^n(x, y, z, \alpha)$. The function $G^n(x, y, z, \alpha)$ is positive over $\mathbf{A_u}$ and negative over $\mathbf{B_u}$ depending on the degree of approximation of $D'(x, y, z, \alpha)$. The convergence of the procedure is analyzed in [8].

The learning points ((x, y, z, α), f(x, y, z, α)) are set with a pseudo random Lp_τ sequence. An important property of the Lp_τ sequences is their optimally spacing, in an arbitrary number of dimensions. As discussed in [11] pseudo random numbers are characterized by enhance convergence when estimating e.g. a multidimensional integral. This defines a priori the number of the learning points in the procedure ($n = 2^p$, 64, 128 or 256,…).

The proposed procedure and its modifications are machine learning [8]. The computer is taught to have the same preferences as the DM. The DM is comparatively quick in learning to operate with the procedure. For example a session with 128 questions (learning points) takes approximately no more than 45 minutes.

Preferences and utility evaluation of the "best" growth rate of the process

The specific growth rate of the fed-batch processes determines the nominal technological condition [7] The complexity of the biotechnological fermentation process makes difficult the determination of the "*best*" process parameters [5, 7]. The incomplete information usually is compensated with the participation of imprecise human estimations. Our experience is that the human estimation of the process parameters of a cultivation process contains uncertainty in the range of 10% to 30%. Here a value-based decision support system for elimination of the uncertainty in the DM preferences and evaluation of the DM's utility is used. The system is mathematically based on the stochastic procedure (5). *The approach permits iterative and precise evaluation of the "best" specific growth rate of the fed-batch process in agreement with the DM's preferences.*

Let ***Z*** be the set of alternatives (***Z***={specific growth rates – μ}=[0, 0.6]h^{-1}) and ***P*** is the convex subset of discrete probability distributions over ***Z***. The expert "preference" relation over ***P*** is expressed through ($\succ$) and this is also true for those over ***Z*** (***Z***$\subseteq$ ***P***). As mentioned above the utility function is defined with precision up to affine transformation (interval scale). The process of utility evaluation and the polynomial utility approximation $U(\mu) = \sum_{i=0}^{6} c_i \mu^i$ are

shown on Fig. 2 and Fig. 3. The utility function is evaluated with 64 learning points and expert answers. This number of questions is for a primary orientation. The seesaw line (Fig. 4) is pattern recognition of $\mathbf{A_u}$ and $\mathbf{B_u}$. This seesaw line recognizes correctly more then 97% of the expert answers. The polynomial approximation of the DM utility $U(\mu)$ is the smooth line (the mathematical expectation).

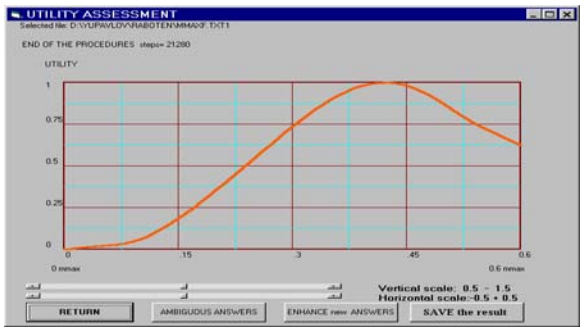

Fig. 2

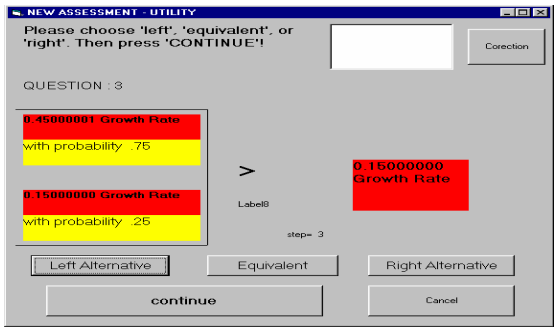

Fig. 3

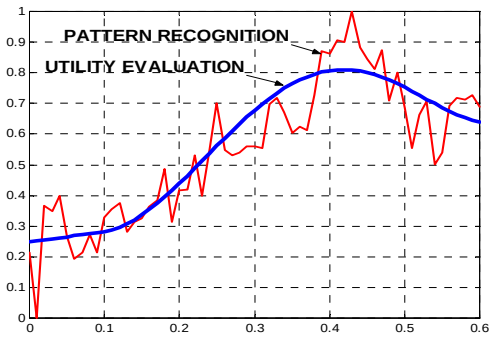

Fig. 4 Utility evaluation of the growth rate

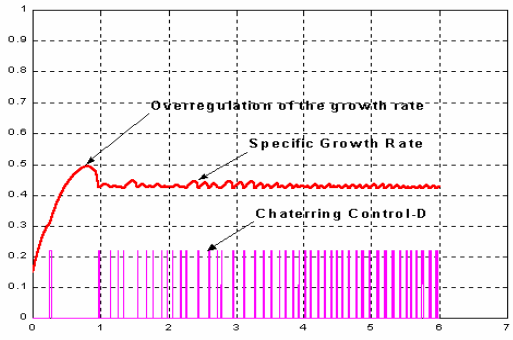

Fig. 5 Stabilization of the growth rate

The expert utility recognizes correctly more then 81% of the expert answers (used learning points). The maximum of the utility function determines the "best" set point of the fed-batch process after the technologist.

Control design and stabilization of the fed-batch process in the "best" growth rate

The polynomial representation of the DM"s utility permits exact analytical determination of the derivative of the utility function and determination of the optimal technological parameters, optimal specific growth rate (optimal set point).

We begin the control design by determination of the Brunovsky normal form of Wang-Yerusalimsky model. On the bases of this equivalent normal form is analytically determined "time minimization" control and chattering robust control stabilization of the growth rate on the "best" utility point.

In the beginning we investigate the Wang-Yerusalimsk*y* model which describes a continuous cultivation process. After that it is shown that the time minimization optimal control of the fed-batch process has the same form as that of the continuous process. The ***continuous process*** is described dynamically by the following model:

$$\begin{aligned}
&\dot{X} = \mu X - DX,\\
&\dot{S} = -k\mu X + (S_0 - S)D,\\
&\dot{\mu} = m(\mu_m \frac{S}{(K_S + S)} \frac{k_E}{(k_{E+} X)} - \mu),\\
&\dot{E} = k_2 \mu E - DE.
\end{aligned} \tag{6}$$

Here D denotes the dilution rate. We apply the next transformation to model (6):

$$\begin{aligned}
u_1 &= \frac{X}{(S_0 - S)}\\
u_2 &= S,\\
u_3 &= \mu,\\
u_4 &= \frac{E}{(S_o - S)}
\end{aligned} \tag{7}$$

The dynamical model (6) obtains the next equivalent form:

$$\begin{aligned}
&\dot{u}_1 = u_3 u_1 - k u_1^2 u_3,\\
&\dot{u}_2 = -k u_1 (S_o - u_2) u_3 + D(S_o - u_2),\\
&\dot{u}_3 = m(\mu_m \frac{u_2}{(K_S + u_2)} \frac{k_E}{(k_E - u_1(S_o - u_2))} - u_3),\\
&\dot{u}_4 = u_4 u_3 (k_2 - k u_1).
\end{aligned} \tag{8}$$

The next step is application of the GS algorithm for exact linearization to Brunovsky normal form, published by Gardner and Shadvwick in 1992 [1, 3]. The new equivalent model of model (6) and model (8) has the form [1, 9, 10]:

$$\begin{aligned}
&\dot{Y}_1 = Y_2\\
&\dot{Y}_2 = Y_3\\
&\dot{Y}_3 = W\\
&\dot{Y}_4 = Y_4 Y_2 \frac{(k_2 - k Y_1)}{Y_1 (1 - k Y_1)}
\end{aligned} \tag{9}$$

The state vector of model (9) has the next explicit extended form:

$$
\begin{aligned}
&Y_1 = u_1 \\
&Y_2 = u_3(u_1 - ku_1^2) \\
&Y_3 = u_3^2(u_1 - 3ku_1^2 + 2k^2u_1^3) + m(u_1 - ku_1^2)(\mu_m \frac{u_2 k_E}{(K_S + u_2)(k_E + u_1(S_o - u_2))} - u_3) \\
&Y_4 = u_4.
\end{aligned}
\tag{10}
$$

The control input of the model (9) is W and it has the next huge analytical form:

$$
\begin{aligned}
W = {} & 2u_3(u_1 - 3ku_1^2 + 2k^2u_1^3)m(\mu_m \frac{u_2}{(K_S + u_2)} \frac{k_E}{(k_E + u_1(S_o - u_2))} - u_3) + \\
& + u_3^3(1 - 6ku_1 + 6k^2u_1^2)(u_1 - ku_1^2) + \\
& + u_3 m(1 - 2ku_1)(u_1 - ku_1^2)(\mu_m \frac{u_2}{(K_S + u_2)} \frac{k_E}{(k_E + u_1(S_o - u_2))} - u_3) - \\
& - u_3 m(u_1 - ku_1^2)^2 \mu_m \frac{u_2}{(K_S + u_2)^2} \frac{k_E}{(k_E + u_1(S_o - u_2))^2}(S_o - u_2) - \\
& - m^2(u_1 - ku_1^2)(\mu_m \frac{u_2}{(K_S + u_2)} \frac{k_E}{(k_E + u_1(S_o - u_2))} - u_3) + \\
& + m(u_1 - ku_1^2) \times \\
& \times \frac{\mu_m k_E(K_S + u_2)(k_E + u_1(S_o - u_2)) - \mu_m u_2 k_E(k_E + u_1 S_o - K_S u_1 - 2u_1 u_2)}{(K_S + u_2)^2(k_E + u_1(S_o - u_2))^2} \times \\
& \times \left[- ku_3 u_1(S_o - u_2) + (S_o - u_2)D \right]
\end{aligned}
\tag{11}
$$

The last equation of model (9) can be solved by separation of variables. Consecutively the variable Y_4 depend only from Y_1 and can be described analytically by $\boldsymbol{Y}_1$. The solution is:

$$
Y_4 = k_4 Y_1^{k_2} \left| 1 - kY_1 \right|^{(1-k_2)}, \quad k_4 \in \boldsymbol{R} \tag{12}
$$

That is why (6), (8) and (9) are dynamically equivalent to the next Brunovsky normal form [1, 3, 10]:

$$
\begin{aligned}
&\dot{Y}_1 = Y_2 \\
&\dot{Y}_2 = Y_3 \\
&\dot{Y}_3 = W
\end{aligned}
\tag{13}
$$

The input D of the continuous model (9) takes part in the last mathematical expression of this equation. Now we can solve the following optimal control problem, where $U(\mu)$ is a unimodal polynomial function, **criteria for optimisation** and control [9, 10]:

$$\mathbf{max}(U(m)), \quad \mu \in [0,\ \mu_{\max}], \quad t \in [0,\ T],\ D \in [0,\ D_{\max}]$$

$$\begin{aligned}
\dot{X} &= \mu X - DX, \\
\dot{S} &= -k\mu X + (S_0 - S)D, \\
\dot{\mu} &= m(\mu_m \frac{S}{(K_S + S)} \frac{k_E}{(k_E + X)} - \mu), \\
\dot{E} &= k_2 \mu E - DE.
\end{aligned} \tag{14}$$

We apply the Pontryagin's maximum principle to the Brunovsky form (13) step by step for sufficiently small time periods T [6]. The control law has the analytical form:

$$D_{opt} = sign\left(\left(\sum_{i=1}^{6} ic_i \mu^{(i-1)}\right)(T-t)\left[\frac{(T-t)\mu(1-2kY_1)}{2} - 1\right]\right) D_{\max} \tag{15}$$

where : $sign(r) = 1, r > 0,\ sign(r) = 0, r \le 0.$

The time interval T is chosen close to the step of discretization of the differential equation solver. The sum in equation (15) is the derivative of the polynomial function $U(\mu)$. It is clear that the "time-minimization" control is determined from the ***sign*** of the derivative of the function $U(\mu)$. Thus, the control is $D = D_{\max}$ or $D = 0$. The solution is a *"time-minimization"* control (if the time period T is sufficiently small). The control brings the system back to the working point for minimal time in the case of growth rate deviations [9, 10].

The previous solution permits easy determination of the control law of ***the fed-batch process***. The control law is based on the solution of the following optimization problem:
Max$(U(\mu(T_{int})))$**, *where the variable* μ *is the specific growth rate,* ($\mu \in$[0, μ_{max}], $F \in$[0, F_{max}]). *Here* $U(\mu)$ *is a unimodal utility function* and *F* is the control input (**the substrate feed rate**)**:

$$\mathbf{max}(U(\mu(T_{\text{int}}))), \mu \in [0, \mu_{\max}], t \in [0, T_{\text{int}}], F \in [0,\ F_{\max}]$$

$$\begin{aligned}
\dot{X} &= \mu X - \frac{F}{V} X, \\
\dot{S} &= -k\mu X + (S_0 - S)\frac{F}{V}, \\
\dot{\mu} &= m(\mu_m \frac{S}{(K_S + S)} \frac{k_E}{(k_E + X)} - \mu), \\
\dot{V} &= F, \\
\dot{E} &= k_2 \mu E - \frac{F}{V} E.
\end{aligned} \tag{16}$$

The control law ***of the fed-batch process*** has the same form (15) because $D(t)$ is replaced with $F(t)/V(t)$ in model (1). Thus, the feeding rate $F(t)$ takes $F(t) = F_{\max}$ or $F(t) = 0$.

We conclude that the control law (15) brings the system to the set point ("best" growth rate) with a "time minimization" control, starting from any deviation of the specific growth rate (Fig. 5).

Thus, we design the next robust control law for attainment of the "best" growth rate and stabilization of the process in this set point [9, 10]:

- Time interval – $[0, t_1]$: the control is a "time-minimization" control (Eq. (15)), where $\mu(t_1) = (x_{30}-\varepsilon)$, $\varepsilon > 0$, $x_{30} = max(U(\mu))$. The input D is replaced with $F = \gamma F_{max}$, $1 \geq \gamma > 0$, when $D = D_{max}$. The choice of γ depends on the step of the equation solver and is not a part of the optimization (in this investigation);
- Time interval - $[t_1, t_2]$: the control is $F = 0$ ($\mu(t_1) = (x_{30}-\varepsilon)$, $\mu(t_2) = x_{30}$ and $d/dt(\mu(t_2))= 0$ (to avoid an over-regulation, (Fig. 5));
- After the moment t_2 the control is the control (15) with $F = \gamma F_{max}$, when $D = D_{max}$ (chattering control with $1 \geq \gamma > 0$).

The performances of the fed-batch process with this control law are shown on Fig. 6. After that the stabilization of the fed-batch process can be maintained around the optimal parameters with a sliding mode control (Fig. 6) [9, 10].

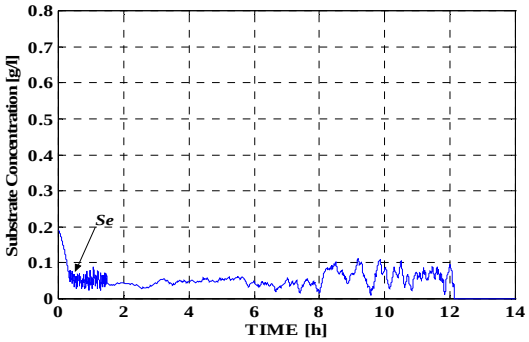

Fig. 6 Optimal profile of the process

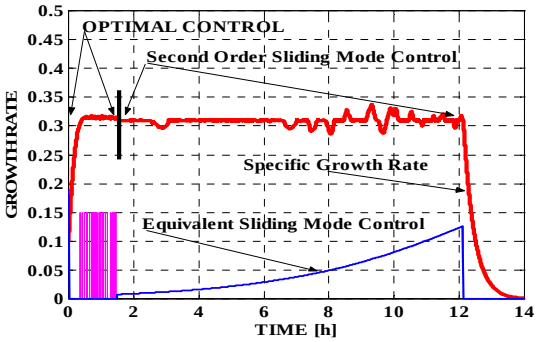

Fig. 7 Substrate concentration S

Interesting moment is the determination of approximations of the moments t_1 and t_2. A manifold is determined and applied for numeric approximation of the moments t_1 and t_2 when the fed-batch process is described by a Wang-Monod kinetic model [12]. We denote with μ_e the growth rate and X_e is the biomass concentration in steady state. The moment t_1 is determined when the state vectors of the Monod model intersects this manifold [9]. The moment t_2 is the moment of intersection of another manifold $(\mu = \mu_e) \cap (d\mu/dt = 0)$. This solution needs determination of the substrate concentration S_e in steady states (the "best" point of the process): The substrate concentration S_e of the Monod model is determined by the equation:

$$\mu_e = \mu_m \frac{S_e}{k_e + S_e} \quad (\text{optimal set point}) \Rightarrow \quad S_e = \frac{K_s \mu_e}{\mu_m - \mu_e} \tag{17}$$

When the Wang-Yerusalimsky kinetic model is used the situation is a litle different. The substrate concentration S_e now depends both from the growth rate μ_e and from the biomass concentration X_e.

$$\mu_e = \mu_m \frac{S_e}{(K_s + S_e)} \frac{k_e}{(k_e + X_e)} \tag{18}$$

The growth rate μ_e and the biomass concentration X_e depend on the moment of intersection with the manifold. A possible way out of this situation is replacement of the biomass concentration X_e with $X(t_1)$ and calculation of the manifold in each step of the equation solver.

$$S_e \approx \frac{K_s \mu_e (k_e + X(t_1))}{(k_e \mu_m - \mu_e (k_e + X(t_1))}. \tag{19}$$

In all cases this will lead to augmentation of the calculations.

Sliding mode control and stabilization in the "best utility" growth rate

The sliding mode (SM) control is realized with Wang-Monod model (20). The solution is obtained with **alternations of the maximum specific growth rate** $\mu_m(t^o, pH)$ through changes of the temperature (t^o) and the acidity of the bioreactor medium (pH).

$$\begin{aligned}
\dot{X} &= \mu X - \frac{F}{V} X, \\
\dot{S} &= -k\mu X + (S_0 - S)\frac{F}{V}, \\
\dot{\mu} &= m\left(\mu_m \frac{S}{(K_S + S)} - \mu \right), \\
\dot{V} &= F, \\
\dot{E} &= k_2 \mu E - \frac{F}{V} E.
\end{aligned} \tag{20}$$

This control choice gives us the possibility for utilization of the temperature (t^o) and the acidity (pH) as control values. More classical SM solutions with substrate concentration S as control value could be seen in the scientific literature. The sliding affine subspace is defined by the equation:

$$Sl_1(\mu) = (\mu - 0.31) = 0 \tag{21}$$

The general stability conditions are derived from the Liapunov's function $(Sl_1)^2$. The equivalent growth rate control is determined exactly and the SM control is possible:

$$U_e \mu = \frac{(K_s + S)\mu}{S} \tag{22}$$

The substrate concentration S is also a constant $S_e = 0.0498$ (Eq. (17)). The feeding rate $F(t)$ is derived from the substrate concentration: $F(t)=(kX(t)\mu(t)V(t)/(S_0 - S_e)$, where $X(.)$ is the quantity of biomass in the bioreactor. The mathematical model and the corresponding stability conditions determine the SM control law:

$$Control \Rightarrow \Delta\mu_m = -\left[\left| \frac{(K_s + S)\mu}{S} - \mu_m^1 \right| + \mu_m^2 \right] sign(Sl_1) \tag{23}$$

The variations of the (t^o) and (pH) assure the chattering of μ_m around the equilibrium ($\mu_m=\mu_{m0}+\Delta\mu_m$), $\mu_m^1 = x_{30}$, $\mu_{m0} = \frac{(K_S + S_e).x_{30}}{S_e}$. The value μ_m^2 is a sufficiently small

supplementary value.

The Russian scientists Emelyanov, Korovin and Levant evolve high-order sliding mode methods in control systems [4]. We propose in our investigation a second order sliding mode control following Emelyanov and Korovin. Out of this approach the second order SM manifold becomes:

$$S_1 \cap \dot{S}_1, \text{where } S_1 = (\mu - 0.31) \; and \; \dot{S}_1 \; is \; the \; time \; derivative \tag{24}$$

Here is used the so-called "contraction" algorithm [4]. After Emelyanov the SM control input in second order "contraction" algorithm becomes:

$$Control \Rightarrow \Delta\mu_m = -\left[\left|\frac{(1.15K_s + 1.15S)1.15\mu}{0.85S} - \mu_m^1\right| + \mu_m^2\right](\frac{2}{3}sign(S_1) + \\ + \frac{1}{3}sign((\mu_m \frac{S}{(S+K_s)} - \mu)(\mu_m - 0.59)) \tag{25}$$

It is well known that this algorithm ends for finite time [4]. The performances of the system with this SM control are shown on Fig. 6. The input in second order SM is smoother but the control is litle more imprecise.

Conclusions

The proposed utility evaluation procedure and its modifications are a machine learning approach. The computer is taught to have the same preferences as the DM in the specific problem. The DM's utility function determines with mathematical exactness the "best" growth rate of the fed-batch process. The following points should be highlighted:

- After understanding the term "lottery", the DM is comparatively quick in learning to operate in the software environment. He is able to answer to the maximum of his abilities. For example, a session with 128 questions takes approximately 50 minutes. In practice, increasing the number of questions adds only little extra time to the dialogue;
- The recurrent stochastic utility procedures are easy to implement in software. The questions in these procedures are homogeneous and require only qualitative answers;
- The Monod kinetic model is a restricted form of the Yerusalimsky kinetic model ($k_E \rightarrow \infty$). The control law is based on measurements of the specific growth rate.

This approach permits iterative preferences based engineer control design. The stochastic utility evaluation and the designed decision support system can be used in complex control peroblems.

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