Techniques to Assess Stationarity and Gaussianity of EEG: An Overview

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Abstract: *Often, while analyzing naturally generated signals (such as speech, seismogram, electroencephalogram, etc.), questions of stationarity and Gaussianity (normality) arise. This paper presents a brief literature overview focused on different methods proposed for assessing stationarity and Gaussianity of electroencephalogram (EEG). Summaries of several methods are discussed.*

Keywords: *Electroencephalogram, Gaussianity, Normality, Stationarity.*

Introduction

Electroencephalography (EEG) is the recording of electrical activity of neurons within the (usually human) brain performed across the scalp [11]. Due to the nature of EEG, its processing often requires detection of non-stationary and non-Gaussian regions. For instance, a spectral analysis can only be performed for stationary data, stationarity, therefore, may be considered as an essential property to assess while processing EEG signals.

To be stationary, the signal's statistical characteristics should not vary over time. In other words, the signal should be time-invariant, which leads to the following conclusion: the average power of a stationary process must be constant over time. To be strict-sense stationary, the process' mean value, standard deviation, the autocorrelation function, and its statistical moments of all orders must not change over time. The latter is rarely possible to verify in practice. More practically, the process is usually defined as weakly (or wide-sense) stationary if only the requirements regarding its mean value and the autocorrelation function are met.

In this report, we present a brief literature overview summarizing various approaches to stationarity and Gaussianity analysis of EEG in their historical retrospective.

Overview of methods

First statistical models of EEG generation were proposed in 1950th suggesting that EEG might be described as a general Gaussian process [22, 24, 36]. The latter might indicate that the sources of EEG activity (neurons) are acting as independent oscillators. As a result, Gaussianity of EEG may be explained using the Central Limit Theorem stating that a sum of a large enough number of independent identically distributed random values tends to have Gaussian distribution. This observation, however, was not confirmed by other authors [15, 16, 25]. Since the EEG data were usually segmented into short fragments for the analysis, the latter disagreement, therefore, might be attributed to dissimilar durations of EEG fragments used by different authors. Thus the question of proper EEG segmentation appeared as important and was extensively studied by researchers since then.

In 1967, Campbell suggested that EEG should be considered as a stationary process [16]. Justifying this assumption, he stated: "Practically, it is not possible to obtain an ensemble of the brain wave process. In fact it is only possible to obtain one realization of the process" [16, 37]. His further remark was to find a general descriptor that can detect any small statistical changes. Campbell also performed several tests to verify the Gaussianity of EEG. He suggested studying first the amplitude distribution of the EEG that may later be compared to the Gaussian distribution. The latter might be achieved, for instance, by the chi-square test. Campbell concluded that 96.7% of thirty 1-minute long fragments he analyzed were not Gaussian: "The brain wave is not a Gaussian process because not even its amplitude distribution is Gaussian" [37].

In his research in 1969, Elul compared two different states of the brain: the idle brain state and the mental task state [31]. He hypothesized that, in the idle brain state, EEG should mostly be Gaussian. Elul proposed dividing EEG data into 2 second-long epochs and comparing the amplitudes of these epochs. He has concluded that, in the idle state, 66% of EEG was Gaussian; while performing mental tasks decreased this figure to 32%. Elul suggested that the amplitude analysis of EEG may provide an important insight on mental activities. Considering the amplitude distribution, any mental task may lead to an observable decrease in Gaussianity [31]. While some later studies agreed with Elul, other authors have questioned his conclusions [17, 20]. Similarly to Campbell, Elul also used the chi-square technique to assess whether the 2-second epochs can be deemed Gaussian. Justifying his choice of 2 seconds for the epoch duration, he wrote: "Although it might appear that a larger sample should contribute to increase confidence, this is true only if the statistical properties of the data are homogenous throughout duration of the entire record (in other words, the data should exhibit stationarity in the wide sense). It has been observed that with data blocks lasting over 2 seconds, the effects of inherent non-stationarity of the EEG become increasingly serious, leading to erroneously low estimates of goodness-of-fit" [31]. Although Elul's method was based on the direct interpretation of Gaussianity, only the amplitude distribution was considered. As proposed later [18], other statistical tests (such as kurtosis excess, skew factor, or correlations comparison) may be implemented to extract Gaussian and stationary fragments with higher confidence.

McEwen, in his work in 1975, has partitioned an EEG signal into two equal epochs. He assumed that, if the original EEG record was wide-sense stationary, the amplitude distributions and the power spectra of these two epochs should not differ significantly. This assumption was then verified by the Kolmogorov-Smirnov test since McEwen generally observed better results while using sample mean and variance than with the chi-square test [5, 10, 17, 23]. According to the author, assessing the amplitude distribution of a set of EEG samples for stationarity can be achieved using a goodness of fit test verifying whether the EEG epochs are wide-sense stationary. McEwen also discussed the importance of sampling frequency: "In practice, as the rate of sampling a band limited EEG segment increases above the Nyquist rate, successive samples become more interdependent and the efficacy of statistical hypothesis tests is consequently affected. It is therefore not surprising that one study of 2 sec EEG segments which were sampled at 200 Hz concluded that resting EEG activity is Gaussian 66 percent of the time, while other studies of EEG segments of similar duration which were sampled at 5000 Hz concluded that resting EEG activity is strongly non-Gaussian" [12, 13, 17, 31].

The effects of increasing sampling rate on the results of Kolmogorov-Smirnov test for Gaussianity are illustrated in Fig. 1 [17] where the initial sampling frequency, F_s, was selected as 64 Hz – slightly above the Nyquist rate.

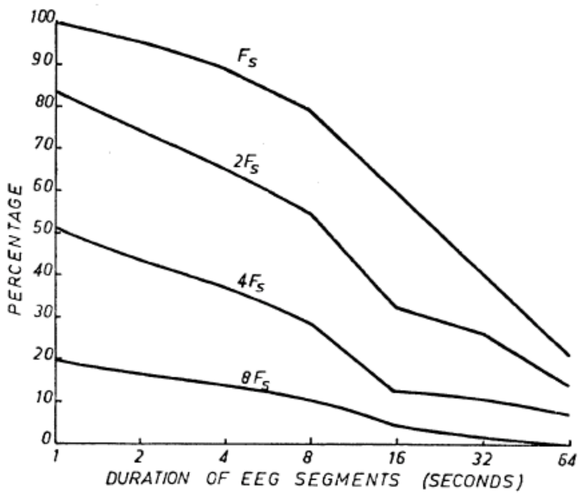

Fig. 1 Effect of increasing sampling rate
on K-S goodness of fit tests for Gaussianity [17]

As seen in Fig. 1, increasing sampling frequency leads to a significant decrease in the percentage of fragments deemed as Gaussian. The critique attributed to the McEwen's approach is the limitation of the sampling frequencies to the range from 64 Hz to 512 Hz since the selection of sampling frequency should be based on the signal's spectrum [19].

In the summary, McEwen concluded that short fragments (up to 10 s) generally follow the normal distribution while longer fragments (up to 60 s) are not Gaussian. He suggested that EEG might be viewed as a process composed from short Gaussian fragments. A fraction of fragments that may be considered as Gaussian reduces from 90% to 20% when the fragments' duration increases from 4 to 60 seconds [17]. On the other hand, up to 90% of 4 second-long fragments may be deemed wide sense stationary while this number reduces to 70-80% when analyzing 16 second-long EEG fragments [17].

In 1973, Kawabata indicated that statistical properties of the EEG sequence should not change over time to ensure stationarity required for the spectral analysis [19]. He defined two conditions for an individual: "eyes closed" and "eyes opened" to compare EEG series. Kawabata assumed that 1-second epochs were definitely stationary [19-21]. He segmented ten 5 second-long EEG signals into twenty 2.5-second fragments for both eyes closed and eyes opened conditions. The mean, variance and power spectra were estimated for each fragment.

Author applied the "run test" and the "trend test" to these 3-parameter sequences formed out of mean, variance, and power spectrum [22].

Kawabata described the run test as follows: "The sequence can be classified into (+) or (-) depending on each value of the sequence. (+) may be given if the value is greater than a constant (e.g., a median of the sequence), otherwise (-) may be given. Thus the sequence of (+) and (-) can be obtained. A run is defined as a sequence of identical observations that are followed or preceded by a different observation or no observation at all." [25]. The trend test can be viewed as an interactive version of the run test where the threshold level is not a constant, but a variable that may be adjusted according to the trend observed in the sequence [29].

For 25 second-long EEG fragments, no rejections from the run and from the trend tests were observed. Therefore, the assessment of non-stationarity based on the run and trend tests indicated that 25 second-long series were locally stationary. Repeating the experiment with twenty 2.5-second fragments (total of 50 seconds of EEG record), 10-20% of the records were rejected by the trend test, while the run test results were unchanged [29]. Two students (22 and 26 years old) were serving as the experimental subjects. Kawabata concluded that EEG of duration up to 25 seconds might be considered as a stationary series since there were very small amount of rejections by any of the two tests (for neurologically normal EEG) [29, 30]. On the other hand, by increasing the duration of the signal parts assumed as locally stationary, Kawabata observed a significant decrease in the stationarity of the EEG signal.

In his paper published in 1977, Cohen presented a brief literature overview and discussed a theory for the estimation of duration of locally stationary fragments [6]. For his experiments, 104 neurologically normal subjects have participated in EEG acquisition. The subjects were relaxed, quiet, and resting on a bed in a darkened room with their eyes closed. Similarly to Kawabata, Cohen also assumed that 1-second epochs were stationary [6, 7, 20]. Evaluating the average amplitude of the 128 epochs, Cohen found medians of different statistical parameters corresponding to these 128 1-second epochs and clustered medians in sequences [6]. Comparing each parameter to its median, he labeled the epochs as positive if the parameter was above the median or negative otherwise. Cohen concluded that, based on the mean value and the frequency structure, EEG epochs up to 12 seconds in duration may be considered as stationary with the negligible error [6, 7]. He also suggested that, based on the behavior of the mean value only, 24 second-long EEG fragments might be assumed as stationary. The later resulted in the probability of error of 10% [6, 7]. Finally, approximately 35% of 64 second-long EEG fragment were deemed as non-stationary [6].

Another approach to the EEG stationarity analysis, an adaptive segmentation proposed by Bodenstein and Praetorius, Jansen, and colleagues in 1977 [2, 14], implemented evaluation of a model of EEG (for instance, autoregressive) for short fragments first. The fragment duration was then incrementally increased (a growing time window was formed) while the model parameters were repeatedly evaluated and compared to their previous estimates. The fragment duration, at which the difference between two sets of parameters exceeded some pre-defined threshold, was deemed the critical duration for stationarity [2, 14].

Jansen in 1981 applied a 5^{th} order autoregressive model to EEG using Kalman filtering technique [3]. Comparing Kalman filtering approach with the DFT-based method of EEG spectral estimation, author suggested that the parametric technique provides more reliable results for estimations in time in the presence of noise. Kalman approach may be viewed as an adaptive method providing high-resolution spectral estimations even for non-complete periods of the waveform [4]. Bohlin has defined Kalman filtering as an estimator for the time varying AR model coefficients [35].

Jansen compared the results obtained with a Kalman filter to the results of the adaptive segmentation previously explored while using the same EEG data [3, 14]. He concluded that the piecewise analysis method is more useful “in extracting elementary patterns from an EEG” then the adaptive segmentation [3].

An alternative approach, referred to as a non-parametric segmentation technique, was developed in 1993-1999. Authors proposed a method of detection of the change-points (boundaries of locally stationary segments) in the EEG time series pre-filtered in the alpha rhythm [1, 8, 32]. A synchrony index was computed for the alpha power using a proprietary software tool and compared to a threshold to assess whether the examined EEG sample should be viewed as a change point. For the 12 normal subjects participating in the study, authors reported the average duration of locally stationary fragments as 0.7-0.8 second depending on the EEG channel. No significant difference between the eyes open and eyes closed conditions were observed [33].

Other methods of EEG stationarity analysis include, for instance, use of smooth localized complex exponentials (SLEX) [34], the generalized state-space approach [21], and the techniques exploring the complexity of EEG series [9]. While classifying nonstationary time series, SLEX derived from Fourier complex exponentials provides functions localized in time and frequency making the transform suitable for the analysis of nonstationary data [26, 34]. This method can also be applied to seismic recordings of earthquake origins or nuclear explosions. The spectral estimate of the signal can be obtained with the SLEX transform and algorithms. Another novel approach to assess non-stationarity of EEG, the state space model [21], assumes that time series may be represented as a set of processes driven by frequency-specific noise sequences. The non-stationarity is accounted by the noise variance that is changing over time [21]. Despite emerging of these new analysis techniques, segmentation of non-stationary signals into locally stationary fragments is still, perhaps, the most frequently used in practice.

Conclusion

Segmentation of non-stationary time series is an established approach used in the analysis of EEG together with other techniques. On the other hand, individual conditions and states may considerably alter the properties of EEG making it non-stationary. Since many practical analysis methods are defined for the stationary signals only, the correct segmentation is critical for an accurate EEG processing.

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