Application of Queuing Theory to Analysis of Waiting Time in the Hospital

Manish Kumar Pandey*, **Dharmendra Kumar Gangeshwer**

Department of Applied Mathematics
Bhilai Institute of Technology
Durg, Chhattisgarh 491001, India
E-mails: manish.parthivi@gmail.com, dk.gangeshwar@bitdurg.ac.in

Corresponding author

Received: October 16, 2022 **Accepted: January 24, 2023**

Published: September 30, 2023

Abstract: *The main problem that healthcare workers face in many hospitals is how long it takes patients to receive services. This tendency is becoming more prevalent, posing a threat to healthcare services. The repercussions of keeping people in a long line for medical care can result in a variety of issues, including death. The many server queuing models were utilized to examine the government hospital's service efficiency in this study. Over two weeks, primary data was collected at the hospital using observation and questionnaire methods to find the queuing model that minimizes patient waiting time. The findings revealed that most of the patients were dissatisfied with the hospital's queue management tactics.*

Keywords: *Queuing theory, Hospital, Queue management, Waiting line.*

Introduction

A Danish Engineer Erlang [7] ran an experiment and observed that the demand for telephone traffic varies. The introduction written by Banks [3] began with a hand-drawn simulation that is explained, as well as modelling concepts and benefits. Sundarapandian [14] built a model in queuing theory and calculated the waiting time for queue lengths. Nkrumah et al. [11] investigated how frustrating it might be for patients to have to wait in a line for a long time before being serviced, resulting in patient dissatisfaction since they are unable to receive the desired quality of treatment. The essential elements of queuing theory and its applications were examined by Shanmugasundaram and Umarani [16].

Odewole [12] researched how technology influences banks' effective service delivery and reduces customers' average waiting time for services in a commercial bank. The model was fitted with data, and the results were computed and examined. Rotich [13] employed a queuing model to determine the optimal waiting time and service cost. Burodo et al. [4] investigated the queuing characteristics at First Bank Ltd's Kaura Namoda Branch utilizing three service efficiency parameters: single, two, and three servers. In the case of the Commercial Bank of Ethiopia, Desta and Belete [6] discovered how queue management influences customer satisfaction. In addition, the impact of queue management in the Bank on customer behaviour was investigated. Delayed healthcare service delivery is a key highlighted concern in healthcare centres, according to Khaskheli et al. [9]. It is caused by the inadequate design of the queue system or its mismanagement. The goal of this study was to determine the optimal number of receptionists and doctors for the study locations to improve the performance of existing outpatient department queue systems.

Kalwar et al. [8] analyzed the contributions to identifying the challenges of healthcare in Pakistan, focusing on the solutions offered by earlier studies as well as the general image of Pakistan's healthcare delivery system. Some references are Afrane and Appah [1], Shastrakar and Pokley [17], Michael [10], Burodo et al. [5] and Suleiman et al. [15]. Like other service-oriented organizations, the hospital industry is growing more competitive. A service provider's ability to deliver prompt service improves patient satisfaction. Furthermore, the apparent financial implications of patients waiting range from man-hours spent waiting when a queue forms to a loss of goodwill that might occur when patients are displeased with a system. This study looked at queuing theory and how many servers are used in hospitals to regulate waiting times.

The study is presented as follows: an introduction, a review of the empirical literature, a description of analysis procedures, an analysis of the results and finishes with possible recommendations.

Methodology

a) Data collection sources: Primary data was gathered in this study by observation and the administration of questionnaires.
b) Sample size and population: In this study, the population of the government hospital was estimated to be around 5000 registered patients. However, a sample size of at least 500 should be considered. In this study, 500 registered patients were employed as a sample size.
c) Technique of sampling: The hospital chosen for this study was chosen using a convenience sampling method.
d) Analyze the data: Percentages and frequencies were used to assess the data collected through the questionnaire. Similarly, data collected through observation was used to create exponential queuing models.

Model of exponential queuing

The following are some of the assumptions that will be made during the research:

a) First-come, first-serve (FCFS) queue management will be considered.
b) Patients' reneging, baulking, and jockeying will not be taken into account in the study.
c) There will also be an endless population source.
d) The system can accommodate an endless number of patients.

Kendall's notation: M/M/1: FCFS/∞ used to express the system's model.

For the model, the following parameters would be examined:

- Let arrival and service rate is λ and μ.
- Factor of utilization $\rho = \frac{\lambda}{\mu}$.
- Chance that there will be no patients in the system $P_0 = 1 - \rho$.
- Chance of having n patients $P_n = (1-\rho)\rho^n$.
- Number of patients in the system on average $L_s = \frac{\lambda}{\mu-\lambda}$.
- Number of patients in line on an average $L_q = \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu}$.
- Time spent in the system on average $W_s = \frac{1}{\mu-\lambda}$.
- Time spent in line on average $W_q = \frac{\frac{\lambda}{\mu}}{\mu-\lambda}$.

M/M/S queuing system:

$P_0 = \left\{1 + \frac{\lambda}{k\mu} + \sum_{n=0}^{k} \frac{(\rho)^n}{!n}\right\}^{-1},$

$L_q = \frac{P_0}{!k}\left(\frac{\lambda}{\mu}\right)^k \rho(1 + 2\rho + 3\rho^2),$

$L_s = L_q + \frac{\lambda}{\mu},$

$W_q = \frac{L_q}{\lambda},$

$W_s = W_q + \frac{1}{\mu}.$

Discussion and findings of the model

Patients' socio-demographic profile

Table 1 shows the socio-demographic characteristics.

Table 1. Patient socio-demographic features

Gender	**Frequency**	**Percentage, (%)**
Male	280	56
Female	220	44
Total	500	100
Age		
≤ 18	85	17
18-35	235	47
35-60	135	27
Above 60	45	9
Total	500	100
Highest educational qualification		
Primary school certificate	79	15.8
Secondary school certificate	112	22.4
Higher secondary school certificate	165	33.3
Graduate and above	144	28.8
Total	500	100
Employment status		
Government servants	94	18.0
Retired employees	63	12.6
Self-employed	202	40.4
Students	70	14.0
Other specified occupations	71	14.2
Total	500	100
Marital status		
Single	60	12.0
Married	343	68.6
separated	12	2.4
Divorced	44	8.8
Widowed	41	8.2
Total	500	100

According to gender-wise respondents, males are 280 (56%) and females 220 (44%) respectively. According to the respondents' ages, 85 (17%), 235 (47%), 135 (27%), and 45

(9%) are under 18 years old, 18 to 35 years old, 35 to 60 years old, and over 60 years old, respectively. According to the patients' educational qualifications, primary 79 (15.8%), secondary 112 (22.4%), higher secondary 165 (33.3%), and graduate 144 (28.8%).

Queue management

Table 2 presents the queue management.

Table 2. Queue management

Statement	Fully agree	Agree	Unsure	Disapprove	Fully disapprove
At the hospital, queues are well-managed.	37 (7.4%)	205 (41%)	24 (4.8%)	194 (38.8%)	40 (8%)
There are obstacles in the way of patients being guided through lineups.	94 (18.8%)	209 (41.8%)	12 (2.4%)	149 (29.8%)	36 (7.2%)
The hospital personnel are quite helpful in assisting patients in line.	68 (13.6%)	106 (21.2%)	17 (3.4%)	223 (44.6%)	86 (17.2%)
Queue discipline follows the FCFS pattern.	87 (17.4%)	114 (22.8%)	13 (2.6%)	226 (45.2%)	60 (12%)
Total	56 (11.2%)	146 (29.2%)	7 (1.4%)	243 (48.6%)	48 (9.6%)

Model of exponential queuing

The study used outpatients from the Government Hospital's Department of Obstetrics and Gynecology to create an exponential model. Observations for Monday (Table 3) and Thursday (Table 4) as days dedicated to hospital consultations were made during the third week of March 2022. The number of patients is denoted as *Np* and the minute arrival rate as *Mar*. Patients spent on service on Monday are shown in Table 5.

Table 3. Patients' arrival on Monday (in minutes)

Np	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mar	3	5	8	12	12	15	22	27	30	32	38	42	45	48
Np	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Mar	49	50	52	56	58	59	61	62	62	64	66	68	69	71
Np	29	30	31	32										
Mar	72	74	76	77										

Table 4. Patients' arrival on Thursday (in minutes)

Np	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mar	4	7	11	13	16	20	27	30	32	35	37	40	42	43
Np	15	16	17	18	19	20	21	22						
Mar	46	48	51	53	55	58	62	64						

Table 5. Patients spent time for service on Monday (in minutes)

Number of patient	Minutes spent on service (CA = cabin/clinical area)			
	CA1	CA2	CA3	CA4
1	5	13	6	11
2	18	20	21	23
3	30	24	28	32
4	34	36	36	44
5	40	44	44	54
6	48	64	54	68
7	54	76	68	74
8	64	88	74	82
9	71	92	80	88
10		102	90	94
11		118	102	104
12		136	106	116
13			116	126
14			132	

Collected on Monday at the consulting rooms is being analyzed:

- The arrival rate of patients on average

  $\lambda = \frac{(5-3)+(8-5)+\cdots\ldots\ldots+(76-74)+(77-76)}{31} = \frac{74}{31} = 2.39$ min/pat (minutes per patient).
- Service rate for CA1 = (18 – 5) + (30 – 18) + … + (71 – 64)

  Average service rate = 66/9 = 7.34 min/pat
- Service rate for CA2 = (20 – 13) + (24 – 20) + … + (136 – 118)

  Average service rate = 123/12 = 10.25 min/pat
- Service rate for CA3 = (21 – 6) + (28 – 21) + … + (132 – 116)

  Average service rate = 126/14 = 9 min/pat
- Service rate for CA4 = (23 – 11) + (32 – 23) + … + (126 – 116)
  Average service rate = =109/13 = 8.38 min/pat
- Finally average service rate of all clinical areas $\mu = \frac{7.34+10.25+9+8.38}{4} = 8.7425$
- Number of servers $k = 4$
- The average of all servers' rates is $k\mu$ = 4(8.7425) =34.97= ~35
- The factor of utilization $\rho = \frac{\lambda}{k\mu} = \frac{2.39}{35} = 0.068$
- When the server is in an idle position

  $P_0 = \left\{1 + \frac{2.39}{35} + \sum_{n=0}^{4} \frac{(0.075)^n}{!n}\right\}^{-1} = 0.93$
- As a result, the probability of having no patients in the system $P_0 = 0.93$
- The anticipated number of patients in the queue $L_q = 0.000036$
- The number of patients who will be in the system when it is fully operational

  $L_s = L_q + \frac{\lambda}{\mu} = 0.000036 + \frac{2.39}{8.7425} = 0.27$
- The time in line that you should expect to wait for $W_q = \frac{L_s}{\lambda} = \frac{0.27}{2.39} = 0.113$ min
- The average time a patient spends in line is 0.113 min

$$W_s = W_q + \frac{1}{\lambda} = 0.113 + \frac{1}{2.39} = 0.53$$

- The average duration a patient spends in the system is 0.53 min.

Patients spent on service on Thursday are shown in Table 6.

Table 6. Patients spent time for service on Thursday (in minutes)

Number of patients	**Minutes spent on service**		
	CA1	**CA2**	**CA3**
1	6	5	8
2	11	9	16
3	18	13	28
4	27	18	32
5	33	24	40
6	37	30	52
7	42	36	60
8	47	44	70
9	54	48	82
10	57	64	91
11	66	72	96
12	76	84	108
13	84	92	116
14	88	98	
15	96	104	
16	102		
17	112		
18	118		
19	126		
20	130		

Collected on Thursday at the consulting rooms is being analyzed:

- Arrival pattern of patients on average

$$\lambda = \frac{(7-4) + (11-7) + \cdots \ldots\ldots + (62-58) + (64-62)}{22} = \frac{60}{22} = 2.73 \text{ min/pat}$$

- Service rate for CA1 = (11 – 6) + (18 – 11) + … + (130 – 126)
  Average service rate = 124/20 = 6.2 min/pat
- Service rate for CA2 = (9 – 5) + (13 – 9) + … + (104 – 98)
  Average service rate = 99/15 = 6.6 min/pat
- Service rate for CA3 = (16 – 8) + (28 – 16) + … + (116 – 108)
  Average service rate = 108/13 = 8.30 min/pat
- Finally average service rate of all clinical areas $\mu = \frac{6.2+6.6+8.30}{3} = 7.034$
- Number of servers k = 3
- The average of all servers' rates = $k\mu$ = 3(7.034) =21.09= ~21
- The factor of utilization $\rho = \frac{\lambda}{k\mu} = \frac{2.73}{21} = 0.13$
- When the server is in an idle position $P_0 = 0.66$
- As a result, the probability of having no patients in the system $P_0 = 0.66$

- The anticipated number of patients in the queue $L_q = 0.0010$
- The number of patients who will be in the system when it is fully operational

$$L_s = L_q + \frac{\lambda}{\mu} = 0.0010 + \frac{2.73}{7.034} = 0.39$$

- The time in line that you should expect to wait $W_q = \frac{L_s}{\lambda} = \frac{0.39}{2.73} = 0.143$ min
- The average time a patient spends in line is 0.143 min

$$W_s = W_q + \frac{1}{\lambda} = 0.143 + \frac{1}{2.73} = 0.51$$

- The average duration a patient spends in the system is 0.51 min.

Conclusion

Observational data from a government hospital was used to conduct queue studies. Patients were usually unsatisfied with the hospital's service quality. This relates to the fact that the hospital is overburdened with patients. Thus, hospital staffs are under pressure and forcing them to discharge patients without conducting a comprehensive treatment, which often results in patient discontent. The findings also revealed that the better service, the more satisfied customers were with the hospital. The other queuing model may be used for the proposed research.

Acknowledgements

The authors are very thankful to the unknown referees whose suggestions have helped in improving the paper.

References

1. Afrane S., A. Appah (2016). Queuing Theory and the Management of Waiting-time in Hospitals: The Case of Anglo Gold Ashanti Hospital in Ghana, International Journal of Academic Research in Business and Social Sciences, 4(2), 34-44.
2. Aziati A. N., N. S. B. Hamdan (2018). Application of Queuing Theory Model and Simulation to Patient Flow at the Outpatient Department, Proceedings of the International Conference on Industrial Engineering and Operations Management Bandung, Indonesia, 3016-3028.
3. Banks J. (1999). Introduction to Simulation, Proceedings of the 31st Conference on Winter Simulation: Simulation – A Bridge to the Future, 1(5-8), 7-13.
4. Burodo M. S., S. Suleiman, Y. Shaba (2019). Queuing Theory and ATM Service Optimization: Empirical Evidence from First Bank PLC, Kaura Namoda Branch, Zamfara State, American Journal of Operations Management and Information Systems, 4(3), 80-86.
5. Burodo M. S., S. Suleiman, G. Yusuf (2021). An Assessment of Queue Management and Patient Satisfaction of Some Selected Hospitals in North-Western Nigeria, International Journal of Mathematics and Statistics Invention, 9(8), 14-24.
6. Desta A. Z., T. H. Belete (2019). The Influence of Waiting Lines Management on Customer Satisfaction in Commercial Bank of Ethiopia, Financial Markets, Institutions and Risks, 3(3), 2521-1250.
7. Erlang A. K. (1909). The Theory of Probabilities and Telephone Conversations, Nyt Tidsskr Mat Ser B, 20, 33-39.
8. Kalwar M. A., H. B. Marri, M. A. Khan, S. A. Khaskheli (2021). Applications of Queuing Theory and Discrete Event Simulation in Health Care Units of Pakistan, International Journal of Science and Engineering Investigations, 10(109), 6-18.
9. Khaskheli S. A., H. B. Marri, M. Nebhwani, M. A. Khan, M. Ahmed (2020). Comparative Study of Queuing Systems of Medical Out Patient Departments of Two Public Hospitals,

Proceedings of the International Conference on Industrial Engineering and Operations Management, UAE, 10-12.
10. Michael O. S. (2020). Performance Modelling of Health Care Service Delivery in Adekunle Ajasin University, Akungba-Akoko, Nigeria Using Queuing Theory, Journal of Advances in Mathematics and Computer Science, 35(3), 119-127.
11. Nkrumah S., F. B. Yeboah, E. Adiwokor (2015). Client Satisfaction with Service Delivery in the Health Sector: The Case of Agogo Presbyterian Hospital, International Journal of Business Administration, 6(4), 64-78.
12. Odewole P. O. (2016). Waiting Lines, Banks' Effective Delivery Systems and Technology Driven Services in Nigeria: A Case Study, International Journal of Finance and Banking Research, 2(6), 185-192.
13. Rotich T. K. (2016). Utility Analysis of an Emergency Medical Service Model Using Queuing Theory, British Journal of Mathematics & Computer Science, 19(1), 1-18.
14. Sundarapandian V. (2009). Probability, Statistics and Queuing Theory, PHI Learning Pvt. Ltd.
15. Suleiman S., M. S. Burodo, Z. Ahmed (2022). An Application of Single and Multi-server Exponential Queuing Model in Some Selected Hospitals of the North-Western Nigeria, Asian Journal of Probability and Statistics, 16(2), 1-9.
16. Shanmugasundaram S., P. Umarani (2015). Queuing Theory Applied in Our Day to Day Life, International Journal of Scientific and Engineering Research, 6(4), 533-541.
17. Shastrakar D. F., S. S. Pokley (2017). Analysis of Different Parameters of Queuing Theory for the Waiting Time of Patients in Hospital, International Conference on Emanations in Modern Engineering Science and Management, 5(3), 98-100.

Manish Kumar Pandey
E-mail: manish.parthivi@gmail.com

Manish Kumar Pandey is a Research Scholar in the Department of Applied Mathematics at Bhilai Institute of Technology Durg under Chhattisgarh Swami Vivekananda Technical University, Bhilai in India. His research interests are in operations research and queuing theory.

Assoc. Prof. Dharmendra Kumar Gangeshwer, Ph.D.
E-mail: dk.gangeshwar@bitdurg.ac.in

Dr. Dharmendra Kumar Gangeshwer is an Associate Professor in the Department of Applied Mathematics at Bhilai Institute of Technology Durg under Chhattisgarh Swami Vivekananda Technical University, Bhilai in India. His research interests are in statistics and operations research. He has more than twenty research publications.

© 2023 by the authors. Licensee Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).