

# Power-line Interference Elimination from ECG Signals Using Notch Filtration: A Quasi-real Time Version

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**Abstract:** Different types of notch filtration have been developed and used for power-line-interference (PLI) suppression in acquired and preprocessed ECG signals; the most of them originate of the distant past. Generally, the traditional notch filters (NF) affect ECG frequency components around the rated PLI frequency. Recently Dotsinsky [1] published a successful off-line notch filtration providing almost total PLI elimination. Recordings taken from the AHA database are mixed by synthesized interference with frequency, which varies from 49 to 51 Hz within 20 s. They are subjected in MATLAB environment to bidirectional band pass (BP) filtration with central frequency  $f_0 = 50$  Hz. Thus, the phase shift is compensated and the zero crossings of the extracted sinusoidal BP waves are identical with that of the PLI. In this study the standard bi-directional BP is replaced with such one working in a quasi-real time thus allowing immediate tracking and interpretation of the ECG signals. A third 150 Hz PLI harmonic is also added as part of the disturbance to bring us closer to real ECG acquisition conditions. Further on, the variable sine wavelengths are measured between two consecutive zero crossings defined as first positive BP sample following a series of negative ones. The inter-sample distances are equated to unity. The left and right lateral distances are calculated using similar triangles and added to the inter-sample distances. The obtained fractal wave periods are converted into corresponding variable PL frequencies  $f_i$ . Corrupted signals are then subjected to bidirectional notch filtration (NF) with narrow stop-band using  $f_i$  as continuously updated central frequency. The carried out experiments show error about  $2 \mu V$  between original and processed ECG signals with sampling rate  $Q = 5$  kHz.

**Keywords:** ECG signals, Power line interference suppression, Bidirectional band pass filtration, Narrow notch filtration, First and third harmonics.

## Introduction

Different types of notch filtration have been developed and used for power-line-interference (PLI) suppression in the acquired and preprocessed ECG signals; the most of them originate of the distant past. Generally, the traditional notch filters (NF) affect ECG frequency components around the rated PLI frequency [6, 9]. Yoo et al. [9] proposed a hardware notch filter, which varies its centre frequency with the PL frequency changes. Digital notch filters with narrow band may overcome to some extent this event but they cannot follow PL frequency deviations allowed by the standard. Moreover, the resulting transient time is unacceptably long, either for

adaptive and non-adaptive notch filters [3, 4], while the introduced distortions in the QRS complex and the subsequent ST segment are significant. Pei and Tseng reported notch filter with transient suppression, which is better than that of the conventional ones [6]. A software implementation of the idea has been studied [2]. The results show that small distortions may be obtained by narrower than 49.9 ÷ 50.1 Hz band-pass filter BP, but with an exclusively long tail, which appears every time the centre filter frequency is corrected by steps to follow gradually the PL frequency change.

Stoyanon et al. [8] presented a real time adaptive filter for PLI suppression. The interference amplitude and the frequency are measured. Lagrange interpolation is applied to detect PLI zero line crossings. Then a current reference signal for the adaptive filter is generated every time the interference frequency, calculated as an average of the last 5 interference periods, differs with more than 0.01 Hz from the last frequency measurement. The primary input of the adaptive filter is the mixture of clean ECG signal and mains interference. The reported filter follows extreme PLI amplitude and PLI frequency change ratios for both 50 and 60 Hz interference. The results show PLI suppression with ringing noise below 25  $\mu$ V for diagnostic ECG with QRS slopes lower than 60  $\mu$ V/ms.

Mihov and Badarov [5] pay attention to the fact that generally the PLI has two harmonics: first and third one that have to be considered when removing the disturbance. The authors recalculate the coefficient of the first harmonic notch filter to obtain the corresponding third harmonic coefficient. Both notch filters are then pipelined on the ECG signal. The experiments carried out with ECG signals taken from the AHA database mixed by synthesized mains interferences with 50 and 60 Hz show errors not exceeding 25  $\mu$ V.

Slimanem and Zaid [7] proposed a real time filter for PL frequency detection and single frequency noise cancellation. The filtering process is based on point-by-point fast Fourier transform with an appropriate length of the analysis window leading to evaluation of instantaneous PLI amplitudes, which are then subtracted from the corresponding values of the noisy signal. The method is applicable even in presence of variable PLI frequency within a bandwidth from 49.8 Hz through 50.2 Hz. The carried out experiments result in clean ECGs. The obtained accuracy is evaluated by signal-to-noise ratio (SNR). The authors report SNR = 217 dB for PLI with constant frequency but a frequency variation of 0.5 Hz decreases drastically the ratio down to 40 dB. Generally, SNR versions for such denoising assessment are not applicable, since the ECG interpretation is based on time-amplitude criteria. The best way to present the potential of any method in this field is to show the 'conditionally clean' input signal, the same but summed up with PLI and the processed signal, followed by the difference between input and processed ones. Still, the published method has a low computational complexity, which makes it suitable for real time cleaning the ECG signals, thus serving for accurate diagnostic conclusions in computer based cardiac system.

Recently Dotsinsky [1] published a successful off-line notch filtration providing almost total PLI elimination. Recordings taken from the AHA database are mixed by synthesized interference with frequency, which varies from 49 through 51 Hz within 20 s. They are used in MATLAB environment to test the method and evaluate the results obtained.

The corrupted ECG signals are subjected to BP filtration with central frequency  $f_o = 50$  Hz, the left and right cut-offs being  $f_l = 48$  and  $f_r = 52$  Hz, respectively. To avoid the phase shift, the BP filter is bidirectional. Thus the extracted sinusoidal BP waves differ in amplitude from the synthesized PLI but their zero crossing points remain identical.

Further on, the variable sine wavelengths are measured between two consecutive zero crossings defined as first positive BP sample following a series of negative ones. The inter-sample distances are equated to unity. The left and right lateral distances are calculated using similar triangles and added to the inter-sample distances. The obtained fractal wave periods are converted into corresponding variable PL frequencies  $f_i$  (Fig. 1).

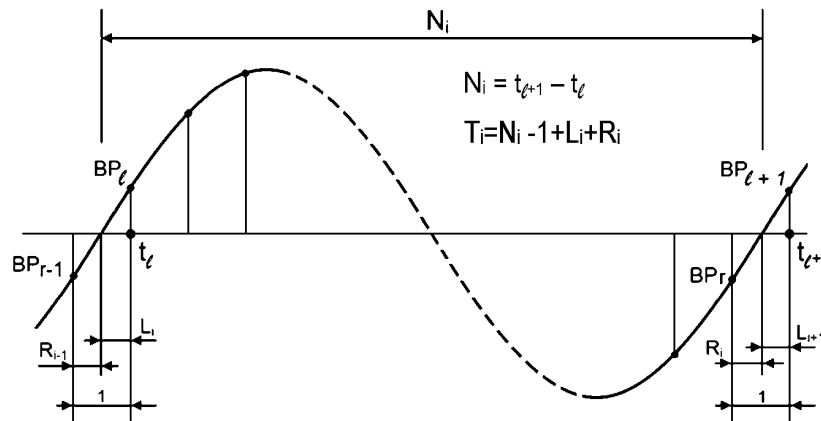


Fig. 1 Variable PLI wavelengths (figure borrowed from the reference [1])

Corrupted signals are then subjected to forward NF with narrow stop-band using  $f_i$  as continuously updated central frequency. The -3 dB cut-off frequencies are  $f_i \pm 0.5$  Hz. The committed low error between original and processed ECG signal is reduced once more by a second backward filtration.

The results obtained demonstrate a very successful PLI suppression. Processed signals with sampling rate  $Q = 5$  kHz are compared to the original ones and the differences are presented in normal and zoomed scales. The maximum absolute errors (except for the edges of the recordings corresponding to the transitional processes) are 7  $\mu$ V for the forward notch filtration and 2  $\mu$ V after the backward one.

Generally, the accuracy of the proposed approach depends of the sampling rate. Higher the SR, shorter the lateral distances that leads to increased accuracy in approximating the ends of the sinusoidal curve with straight lines. Experiments with the same signals re-sampled down to 1 kHz, 500 Hz and 250 Hz show errors committed after bi-directional filtration near to zero, 3  $\mu$ V and 19  $\mu$ V, respectively.

The aim of the study is to improve the bidirectional BP procedure and to introduce bidirectional NF so that they could work on a quasi-real time. Another task is to include the 3<sup>rd</sup> PLI harmonic to the synthesized disturbance that will bring us closer to real ECG acquisition conditions.

### Materials and methods

The used ECG signals are the same as in the basic article [1]: original and re-sampled up to 5 kHz recordings. Three of them are taken from the AHA database. The signal ECGprobe is artificially synthesized to reproduce a “normal” form of an ECG signal with heart rate of 60 beats per minute. The signal D0145 is taken from own database. The ECG signals are additionally corrupted by synthesized PLI with variable frequency from 49 through 51 Hz. The amplitude of the PLI is 1 mV for the 1<sup>st</sup> harmonic and 0.1 mV for the 3<sup>rd</sup> one.

The aided third harmonic can be seen together with the AHA1003d1 signal in Fig. 2, where the upper plot represents the amplitude spectra of the mixed signal, which include the conditionally clean input signal as well as the synthesized 1<sup>st</sup> and the 3<sup>rd</sup> PLI harmonics. The lower plot shows the spectra of processed signal. The filtration algorithm is shown in Fig. 3.

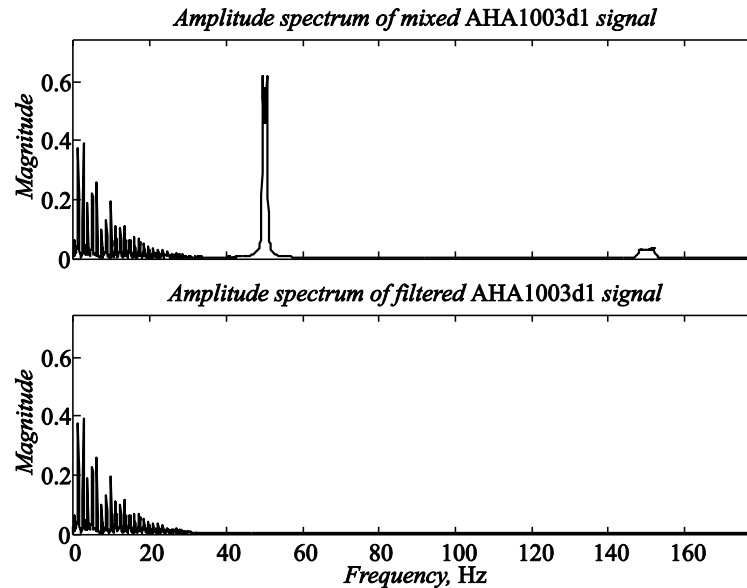


Fig. 2 Upper plot – the included 3<sup>rd</sup> PLI harmonic to the synthesized disturbance; lower plot – the amplitude spectra of filtered signal.

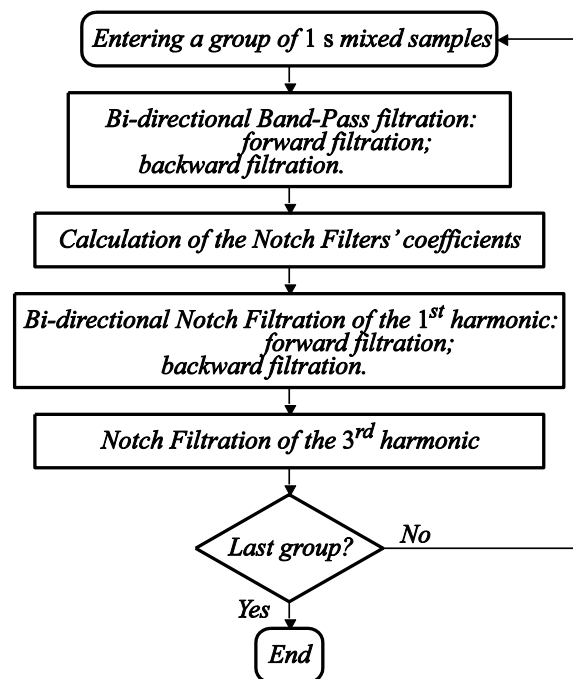


Fig. 3 Flow chart of the filtration algorithm

The bidirectional BP procedure consists of consecutive filtrations performed on signal intervals having different lengths depending on the forward or backward direction. Transfer functions  $H_{BPf}$  and  $H_{BPb}$  of BP forward and backward filters respectively in  $z$ -plane are presented by equation:

$$\left. \begin{aligned}
 H_{BPf}(z) &= \frac{(z^0 - z^{-2}) \cdot k}{z^0 - a_1 \cdot z^{-1} + a_2 \cdot z^{-2}} \\
 H_{BPb}(z) &= \frac{(z^0 - z^2) \cdot k}{z^0 - a_1 \cdot z^1 + a_2 \cdot z^2}
 \end{aligned} \right\} ; \quad \begin{aligned}
 a_1 &= \frac{2}{1+k} \cdot \cos \frac{2 \cdot \pi \cdot f_o}{Q} \\
 a_2 &= \frac{1-k}{1+k}
 \end{aligned} , \quad k = \operatorname{tg} \frac{\pi \cdot (f_r - f_l)}{Q} . \quad (1)$$

Equivalent frequency response of bi-directional filtration is shown on Fig. 4. For the whole frequency axis the phase response retains value of 0 rad.

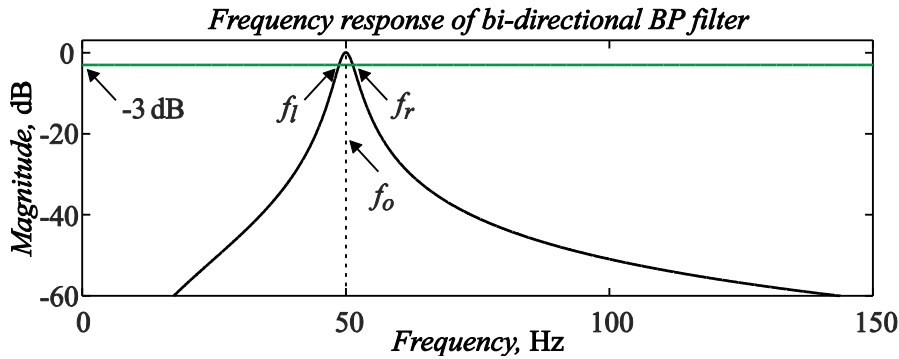


Fig. 4 Frequency response of BP filtration

The principle of the introduced quasi-real time bidirectional BP filtration is shown in Fig. 5. The incoming mixed samples are grouped in intervals of 1 s. Each of them is forward filtered. Then its samples are backward filtered together with that belonging to the second half of the preceding forward interval. The introduced noise illustration at the beginning of the backward arrows (the *stB* intervals) demonstrates the reduced PLI due to the bidirectional procedure.

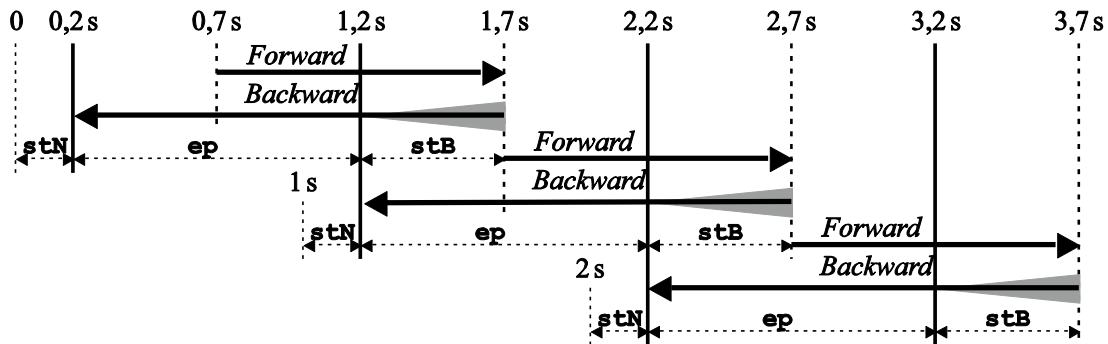


Fig. 5 Principle of the bidirectional BP filtration

A written MATLAB program implements the BP filter, which is based on **a1** and **a2** coefficients calculated using the following parameters:

```

Q = 5000; % sampling rate, Hz
fl = 48; fo = 50; fr = 52; % left, central and right cut-off
                                frequencies
k = tan(pi*(fr-fl)/Q);
a1 = 2*cos(2*pi*(fo/Q))/(1+k); a2=(1-k)/(1+k); % BP coefficients
k_n = tan(pi*(fr-fl)/Q/2); A2=(1-k_n)/(1+k_n); % NF coefficients
    
```

Here **a1** is function of the variable frequency **f<sub>o</sub>**, while **a2** remains constant during the filtration.

The quasi-real bi-directional **BP** filtration is completed by the next intervals and instructions:

```

ep = round(Q); % corresponding to 1 s
stB = round(Q/2); % corresponding to 0.5 s
stN = round(Q/5); % corresponding to 0.2 s
% ep+stB+stN forward BP processed interval
% stN backward BP processed interval

□ for j = 1:ep:length(mix)-ep; % parts for quasi-real processing
| % % forward band-pass filtration
□ for i = j+stB+stN+2:1:j+ep+stB+stN+2
| BPf(i) = a1*BPf(i-1)-a2*BPf(i-2)+(mix(i)-mix(i-2))*k;
| end
| % % backward band-pass filtration
□ for i = j+ep+stB+stN:-1:j+stN+2
| BP(i) = a1*BP(i+1)-a2*BP(i+2)+(BPf(i)-BPf(i+2))*k;
| end

```

The variable period **T<sub>i</sub>** of power-line frequency is estimated according the reported in [1] procedure. The corrupted signal **mix** is processed by twice narrower NF { **(f<sub>r</sub> - f<sub>l</sub>) / 2** } using the detected variable central frequency **f<sub>i</sub> = Q/T<sub>i</sub>** (see the Fig. 1). The coefficient **A1(i)** is a function of **f<sub>i</sub>**, **A2** remains constant.

$$f_i = Q/T_i; A1(i) = 2 \cdot \cos(2 \cdot \pi \cdot (f_i/Q)) / (1 + k_n);$$

The next equations present the transfer functions **H<sub>NFf</sub>** and **H<sub>NFb</sub>** of forward and backward NF whose equivalent bi-directional frequency response for **f<sub>i</sub> = f<sub>o</sub>** is shown on Fig. 6.

$$\left. \begin{aligned}
 H_{NFf}(z) &= \frac{\frac{1+A_2}{2} \cdot z^0 - A_{1(i)} \cdot z^{-1} + \frac{1+A_2}{2} \cdot z^{-2}}{z^0 - A_{1(i)} \cdot z^{-1} + A_2 \cdot z^{-2}} \\
 H_{NFb}(z) &= \frac{\frac{1+A_2}{2} \cdot z^0 - A_{1(i)} \cdot z^1 + \frac{1+A_2}{2} \cdot z^2}{z^0 - A_{1(i)} \cdot z^1 + A_2 \cdot z^2}
 \end{aligned} \right\} \begin{aligned}
 A_{1(i)} &= \frac{2}{1+k_n} \cdot \cos \frac{2 \cdot \pi \cdot f(i)}{Q} \\
 A_2 &= \frac{1-k_n}{1+k_n}
 \end{aligned} , k_n = \operatorname{tg} \frac{\pi \cdot (f_r - f_l)}{2 \cdot Q} \quad (2)$$

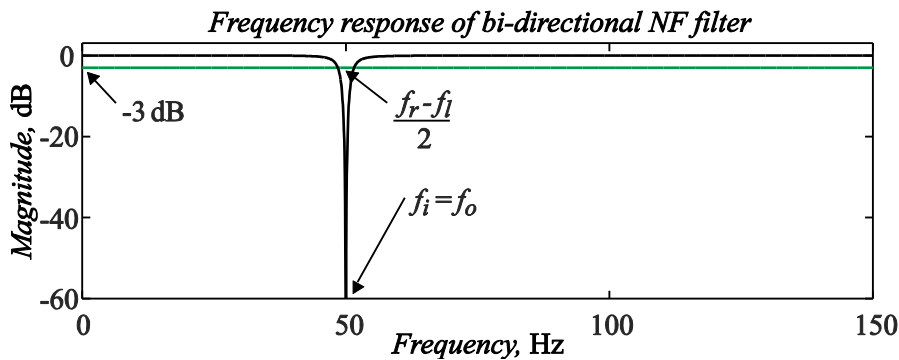


Fig. 6 Frequency response of BP filtration

The principle of the quasi-real bidirectional NF filtration is similar to the BP one. The forward intervals are equal to 1 s, the backward ones have a length of 1.2 s and start from the end of their preceding intervals.

```
% % forward notch filtration of 1-st harmonic
for i = j+stN+2:1:j+ep+stN+2
    NFf(i) = A1(i)*NFf(i-1)-A2*Nf(i-2)+.5*(1+A2)*mix(i) -
            -A1(i)*mix(i-1)+.5*(1+A2)*mix(i-2);
    NFb(i) = NFf(i);
end
% % backward notch filtration of 1-st harmonic
for i = j+ep+stN:-1:j+2
    NFb(i) = A1(i)*NFb(i+1)-A2*Nf(i+2)+.5*(1+A2)*NFf(i) -
            -A1(i)*NFf(i+1)+.5*(1+A2)*NFf(i+2);
    NF(i) = NFb(i);
end
```

The notch filtration of the 3<sup>rd</sup> harmonic is applied just forward at interval of 1 s without overlapping. The variable coefficient  $A3(i)$  is calculated directly from  $A1(i)$ .

```
% % forward filtration of 3-rd harmonic
for i = j+ep+stN:-1:j+2
    A3(i) = A1(i)*(4*(A1(i))*((1+k_n)/2))^2-3
    NF(i) = A3(i)*NF(i-1)-A2*NF(i-2)+
            +.5*(1+A2)*NFb(i)-A3(i)*NFb(i-1)+.5*(1+A2)*NFb(i-2);
end
end
```

## Results and discussion

Fig. 7 illustrates the significance of the results achieved. The upper plot shows the AHA1003d1 ECG signal, the synthesized PLI and the mixed signal, which is subjected further on filtration. The lower plot confirms the extremely successful PLI suppression in ECG signals by means of the developed quasi-real time applicable algorithm for notch filtration. The processed signal AHA1003d1 is compared to the original and the differences are presented in normal and zoomed scales. The maximum absolute error except for the edges of the record (corresponding to the transitional processes) is 0.002 mV.

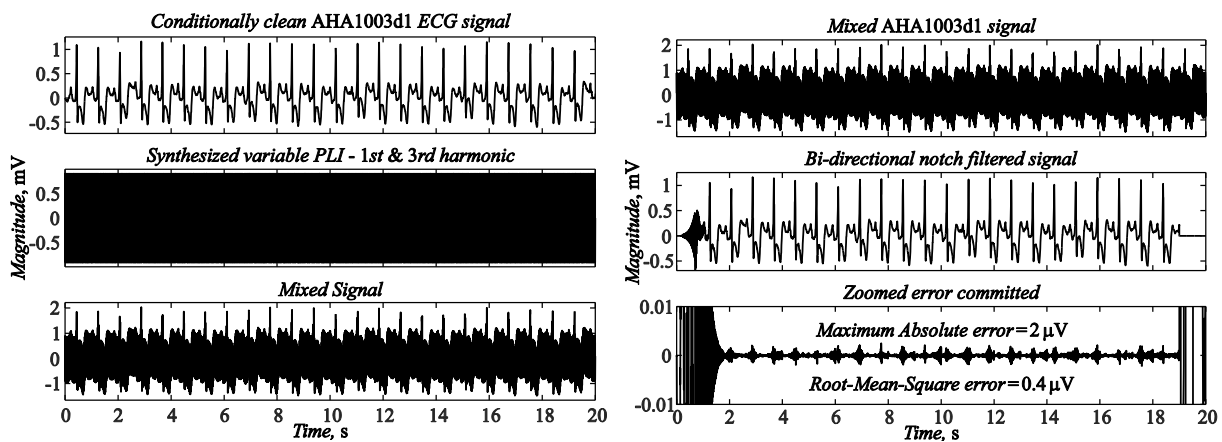


Fig. 7 Left plot – ECG signal, synthesized PLI and the corresponding mixed signal, subjected to processing; right plot – successfully processed contaminated ECG signal.

Fig. 8 demonstrates high results obtained with other ECG signals.

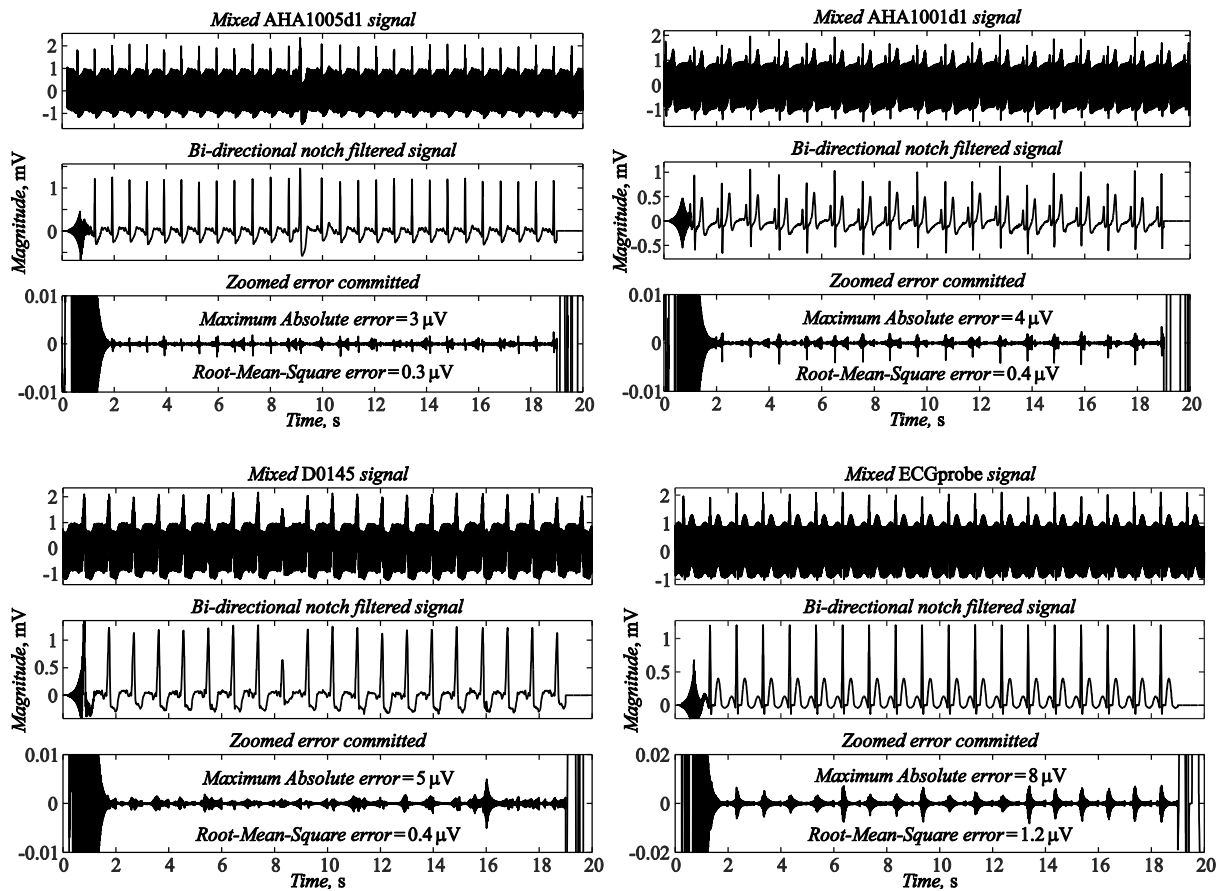


Fig. 8 Results obtained with other ECG signals

Two types of errors are evaluated. The lower subplots show the first of them calculated as difference between conditionally clean and processed signals. One may observe that it reaches maximum values of 2 through 8  $\mu\text{V}$  near to the R peaks, where the ECG frequency components are close to the PLI frequency. Then Root Mean Square errors were determined within the intervals from 2 through 18 s, which do not include the transitional processes. The following table summarizes the errors committed.

Table 1. Errors committed in the filtering process

ECG test signal	Maximum absolute error, $\mu\text{V}$	Root mean square error, $\mu\text{V}$
AHA1003d1	2	0.4
AHA1005d1	3	0.3
AHA1001d1	4	0.4
D0145	5	0.4
ECGprobe	8	1.2

Tests performed with other signals also show identical results.

### Conclusions

The introduced quasi-real bidirectional BP and NF filtration significantly speed up the discussed approach, thus allowing diagnostic interpretations in computer based cardiac system.



The main problem in implementing these filtrations consisted of the overlap time determination, indicated by  $stB$  in Fig. 4 for the BP filter and by  $stN$  in the MATLAB script for the NF filter. This is the time after which the fluctuations caused by the infinite impulse response of the backward filtration attenuate. After numerous tests with different signals, the overlap times of the BP and NF are  $stB = 0.5$  s and  $stN = 0.2$  s, respectively.

The aided third harmonic makes the disturbance simulation closer to real cases.

The used for testing interference is with variable frequency of 0.1 Hz/s that exceeds the real cases. Its amplitude is also higher than normal. These parameters are selected to highlight the high accuracy obtained.

The obtained results point out an extremely successful PLI suppression in ECG signals. Thus, the common on ineffective hardware and software notch filtrations can be replaced by a method for signal processing without any time and amplitude corruptions.

As a disadvantage of the method can be mentioned the non-processed edges of the recordings due to the transitional processes.

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